Economics of Information Networks

SAMPLE Examination Questions and Answers

November 15, 2018

General instructions / 一般説明

Note: this is not a representative sample. These questions are all possible, but there are many others. You should not conclude anything about the relative likelihood of questions from the proportions of topics in this sample. Many questions are not related to the midterm examination.

＊＊注意：代表的なサンプルでなく、試験に問われる可能性だけである。他の種類も可能である。サンプルでの頻度で確立を推計しないこと。＊＊

The instructions below are those used on actual past examinations. If point count is unspecified, I haven't thought about how many points that question would be worth.

＊＊以下の説明は実際の試験で書いたものによく似ている。ただし、点数が指定していない場合には何点はまだ考えていないだけです。＊＊

Japanese translations may follow the English text. If the English and Japanese versions of any text differ in meaning, the English text is correct. Please ask for clarification if you have any doubt. Any corrections will be posted.

英文に次いで和訳がある場合がある。英文と和文の間に食い違いがあれば、英語の方が正しい。ただし、不明な点については遠慮なく聞いてください。あらゆる訂正は掲示します。

Be sure to write your name and student ID number on each sheet.

Several problems in the Economics of Information Networks are presented below. **You may answer in Japanese or English.** In Japanese, please take great care in writing kanji. Avoid abbreviated kanji; the only one I know is the 3-stroke mongamae.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited. Some dictionaries may be provided at the examination.

As usual, the only items that should be present on your desk are pencils, pens, erasers, pencil sharpener, watch, and the examination paper. Put other items in your bag and place the bag under the seat or desk, or on the seat next to you.

Except for calculations, I can give complete answers to most problems within 3 lines. Some questions can be answered correctly with 2 or 3 words. Students usually more space to express their answers, but you should try to avoid making these problems harder than I intend them to be. Answers will be evaluated on presence of the correct idea, not the quantity of words used.

Below each problem enough space is provided for a complete answer. Please write your answers there. If you need more space for an answer, use any available empty space. Clearly indicate where the additional text is, and label it clearly with the question it answers. A figure with axes is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used, and if needed, how you derived them.

General instructions / 一般説明

名前と学籍番号を忘れずに各ページに記入してください。

以下に情報ネットワークの経済学の知識を検定する問題のすべてに解答せよ。**解答の言語は日本語でも英語でも構わない。**もし日本語で書けば漢字などの書き方に十分注意してください。たとえ、省略した漢字などを使わないこと。（私が読めない場合には省略した文字を「間違え」と採点する。）

ノート・教科書・辞書・電卓・携帯電話・その他のメモリを持つ電子製品の用は禁止である。全ての計算は簡単であるので電卓などは必要ない。

机の上にペン・鉛筆・消しゴム・鉛筆削り・時計・この試験用紙の他の物を置かないこと。その他のものを側の席に置くこと。

計算問題以外の問題には私は３行以内十分に答えられる。少数の問題は６文字でも可能だ。もちろん学生の方がより多くのスペースを要するが、私の意味より難し過ぎることを考えないでください。採点ははっきり正しい概念が含まれているかどうかによって判断し、それ以上に文字の数は構わない。

各問題の下に十分にスペースを用意するのでそこに買いてください。それ以上のスペースが欲しかったらどの空白でも構わない。ただし、用意したスペースに「追加あり」を示し、追加のテキストにどの問題の答えかをはっきり表すこと。グラフ問題には軸を用意するのでそれを使用すること。計算問題には結果だけは少数点（０可）しか与えなく、使用した式などが必要だ。場合により、導き出す方法も表すこと。

Problems / 問題

1. `[Problem ID #1] DESCRIPTION`
   In Shy's dynamic analysis of technology choice, young people receive externalities from the previous generation if technology matches. This appears in the utility function $U(T, N)$, where the size of the network at time t is $N = N_t + N_{t-1}$ when technology matches. However, this does *not* account for matches and mismatches when the young become old in the next period.

   "Tell a story" about how these network externalities might be zero, so that Shy's analysis is correct in assuming that u is independent of next the generation's choice. Or, if you believe that is impossible, explain why.

   (**Note for 2017:** You don't need know anything more about Shy's model to answer this question.

2. `[Problem ID #2] DESCRIPTION`
   Here is part of the description of the network in the Braess Paradox.

   Consider 4000 cars trying to get from A to B. There are two routes, path ADB and path ADC. $C \to B$ and $A \to D$ are bridges of small capacity such that the time to cross increases with the number of cars trying to cross the bridge. We denote the number of cars crossing A to D and continuing to B (path ADB) as x, and the number traveling A to C and crossing C to B (path ACB) as y. The bridges have the same capacity, and the time to cross is the same: $x/100$ for ADB, and $y/100$ for ACB. The other parts of the routes are high-capacity highways taking 40 minutes to traverse.

   Why do we give the numbers traveling for the paths rather than the links?

3. `[Problem ID #3] DESCRIPTION`
   Here is part of the description of the network in the Braess Paradox.

   Consider 4000 cars trying to get from A to B. There are two routes, path ADB and path ADC. $C \to B$ and $A \to D$ are bridges of small capacity such that the time to cross increases with the number of cars trying to cross the bridge. We denote the number of cars crossing A to D and continuing to B (path ADB) as x, and the number crossing C to B and continuing to B (path ACB) as y. The bridges have the same capacity, and the time to cross is the same: $x/100$ for ADB, and $y/100$ for ACB. The other parts of the routes are high-capacity highways taking 40 minutes to traverse.

   Now suppose that a new bridge of very high capacity is built between D and C. In the lecture, it takes 0 time to cross the bridge. Suppose instead it takes t minutes, independent of traffic, to cross the new bridge. Discuss how the analysis of the problem changes, if it does. You need to consider different values of t, as there is more than one answer depending on t.

4. `[Problem ID #4] DESCRIPTION`
   In class we discussed analyzing a large weighted network by deleting links in order of increasing or decreasing link strength.

What do you think would happen in case of deletion in order of decreasing neighborhood overlap? Increasing order?

5. `[Problem ID #5] DESCRIPTION`
   Consider the concept of "complete graph."

   (a) Give the definition of *complete graph*.

   (b) Give the definition of *multigraph*.

   (c) Does the concept of "complete multigraph" make sense? If so, give a definition. If not, explain why not.

   (d) In the lecture, it's mentioned that the idea of triadic closure arises for *connected* subsets. Why are connected subsets important?

6. `[Problem ID #6] DESCRIPTION`

   In a network, suppose that each user i has a random value of being connected to each other user j, $u_i(j)$, and the values are independently and identically distributed. Suppose that users are added to the network in a random sequence.

   (a) Does Metcalfe's law hold?
   (b) If so, prove it . . .
   (c) . . . and suggest generalizations where it still holds.
   (d) If not, give a counterexample . . .
   (e) . . . and suggest restrictions that might make it hold.

7. `[Problem ID #7] DESCRIPTION`
   Mathematicians, like game theorists, like to give their examples cute names that are memorable. Explain the following names (or labels) for graphs shown in Lecture 1.

   (a) "Jealousy" (slide 27).
   (b) "Infidelity" (slide 28).

8. `[Problem ID #8] DESCRIPTION`
   Give two examples of networks you participate in or use. For each example, describe the network as a graph:

   (a) What are the objects?
   (b) What are the links?
   (c) Is the graph directed? Explain why or why not.
   (d) Is it colored? What do the "colors" represent.
   (e) Is it a multigraph? Explain why or why not.
   (f) Do the objects have important attributes (other than their links)? What are they?

9. `[Problem ID #9] DESCRIPTION`
   In class, we discussed how the *game of pure coordination* and the *prisoners' dilemma* could be interpreted as models of technology adoption in markets with network externalities. Now let's consider the *asymmetric* game *battle of the sexes*, with payoffs

	New	Old
New	$4, 1$	$0, 0$
Old	$0, 0$	$1, 4$

Table 1: Battle of the sexes

   (a) What are the "good" and "bad" outcomes of this game?
   (b) How is this game similar to the *game of pure coordination*?
   (c) How is this game different from the *game of pure coordination*?
   (d) Is it useful to compare this game to the *prisoners' dilemma*? If so, what lessons do you draw from the comparison? If not, why isn't it useful for comparison?
   (e) What are the equilibria of this game?
   (f) "Tell a story" about how these payoffs might arise if two *end users* of the technology are playing the game.
   (g) "Tell a story" about how these payoffs might arise if two *providers* of the technology are playing the game.

   Note: your answer to the last two questions may be "I can't tell a sensible story." If so, describe your difficulty.

10. `[Problem ID #10] DESCRIPTION`
    Is a cycle strongly connected? Explain your answer.

11. `[Problem ID #11] DESCRIPTION`
    Metcalfe's Law characterizes the value of a network in terms of the number of nodes in the network.

    (a) State Metcalfe's Law.

    (b) Describe a model, with any necessary assumptions, in which Metcalfe's Law holds.

(c) Derive Metcalfe's Law in that model.

12. `[Problem ID #12] DESCRIPTION`
Consider the measures embeddedness and neighborhood overlap for links.

  (a) Define *embeddedness*.

  (b) Define *neighborhood overlap*.

  (c) What is the relationship between these two indicies?

  (d) How are these measures related to the idea of a *bridge*?

13. `[Problem ID #13] DESCRIPTION`
There are two useful notions of "bottlenecks" in a graph.

  (a) Define *gatekeeper*.

  (b) Define *bridge*.

  (c) Can you say anything special about the nodes at the ends of a bridge in the context of "bottlenecks"?

(d) Can you say anything special about the links that meet at a gatekeeper in the context of "bottlenecks"?

14. `[Problem ID #14] DESCRIPTION`
In the analysis of Facebook member's neighborhood of friends, two individuals in the network had a "one-way communication" link if only one sent messages to the other, and a "two-way communication" link if each had sent messages to the other, in a particular one-month period.

    (a) Is there a clear difference in the strength of links between "one-way" and "mutual" links? Explain why you give that answer.

    (b) The researchers defined a third kind of link, a "maintained relationship" where one member checked content on another's Facebook page. This is a one-way link. Do you think it would be useful to also define a "mutually maintained relationship"? Explain your answer.

15. `[Problem ID #15] DESCRIPTION`
For most students (from elementary school to graduate school), their relationships with other students form through several mechanisms: *commuting* together, *club* activities, *class* attendance, *zemi*, and all *other*.

    (a) In your opinion, are there other mechanisms (one or more) as important as commuting, class, club, or zemi that should be explicitly mentioned (not left as "other")? If so, what is it (are they)?

    (b) In your opinion, is there a natural ranking of the strength of interpersonal ties formed through the mechanisms described above (including any you added in part 15a)? (The ranking may be partial.)

    (c) Suppose you had access to the social network activity of all classmates, neighbors, club members, *etc.* of a given person. How would you try to confirm the hypothesis above?

16. `[Problem ID #16] DESCRIPTION`
    (PROBLEM TYPE) For a graph of about 7 nodes (definitely 10 or less) determine whether it is triadic closed (or strong triadic closed). Some examples:

    (a) A simple path of length 5. Is it triadic closed?

    (b) A simple path of length 6, all links weak. Is it strong triadic closed?

    (c) A simple path of length 6, alternating weak and strong. Is it strong triadic closed?

    (d) A square. Is it triadic closed?

    (e) A square. How do you make it triadic closed?

17. `[Problem ID #17] DESCRIPTION`
    In a graph (to be provided) identify all local bridges and local gatekeepers, and also those which are (global) gridges and gatekeepers.

18. `[Problem ID #18] DESCRIPTION`
    In a graph (to be provided), determine the connected components.

19. `[Problem ID #19] DESCRIPTION`
    In a graph (to be provided),

    (a) find all simple paths from A to Z;

    (b) find all shortest paths; and

    (c) find the length of a shortest path.

    (d) Explain why part 19a specifies *simple* paths.

20. `[Problem ID #20] DESCRIPTION`
    It is often useful to consider networks that are "almost" connected as if they were connected.

    (a) Define *giant component* formally (there is no formal or quantitive definition).

    (b) Give an example of a real network that you believe contains a giant component, and explain why you expect a giant component in that network.

21. `[Problem ID #21] DESCRIPTION`
    Breadth-first search is a useful network algorithm.

    (a) Draw a graph containing 7 nodes and 12 links. Label the nodes A, B, …, G. (Feel free to place the nodes arbitrarily and draw crossing links if you need to. Links need not be drawn as straight lines.)

(b) Explain step by step how to use breadth-first search to find the distance from A to each other node. You almost certainly should draw a new graph.

(c) Don't forget the distance from A to itself!

22. `[Problem ID #22] DESCRIPTION`
What is the "small-world phenomenon"? You should describe Milgram's experiment and explain the significance of the number "6".