Economics of Information Networks

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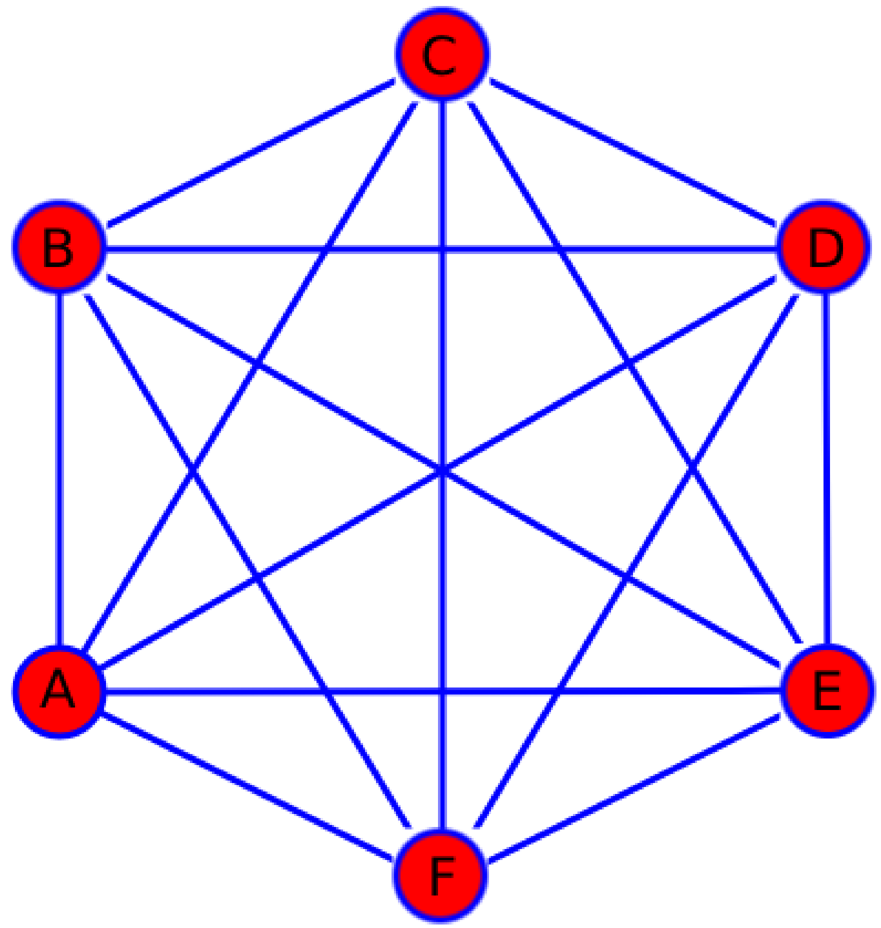
Abstract

We now look at the simplest interesting network model, using the simple "star" market with a network externality. We derive Metcalfe's law and the logistic growth model.

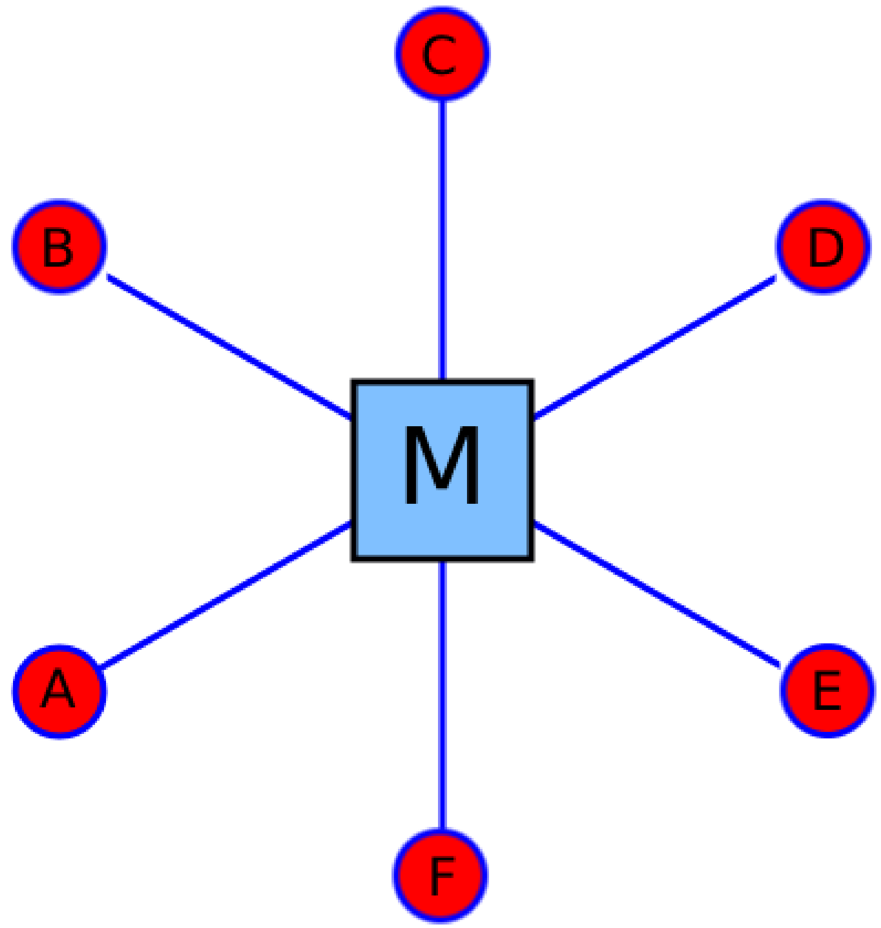
Network Structure of Markets

The simplest network model of a market simply connects traders to each other.

- No distinction between buyers and sellers, or between buying and selling.
- All traders are "in the same place"—as a network, all connected to each other.



An Alternative Model of the Market



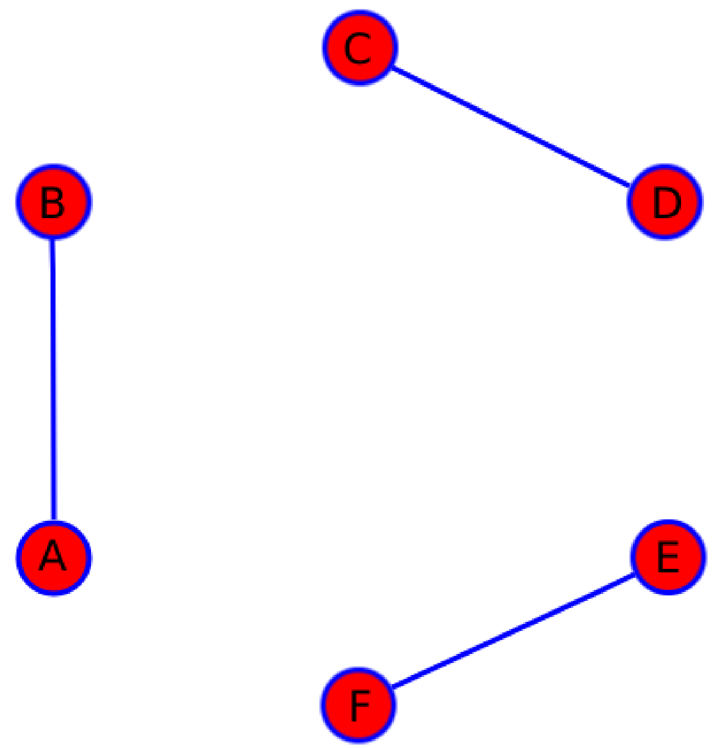
Another simple model connects the traders *through* the market.

The Logic of the Star Market

The links in the two market models are *symmetric*. It's easier to see what this means if we consider a much simpler structure: a set of *barter* relationships.

It's clear that something is missing from the symmetric link: it doesn't provide a way to talk about *equilibrium*, that is, the balance of value given for value received. Symmetry *assumes* balance.

The star arrangement allows the market to assure that balance, and achieve efficiency. Barter won't be efficient because *each* individual trade must balance.

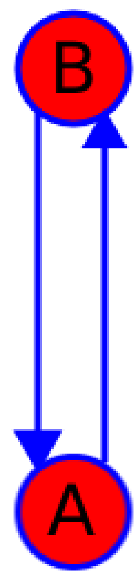


A Directed Graph Approach

Consider one pair in isolation, with directed links indicating transfer of a value from one to the other.

If one link were missing, then balance could not be achieved and the transaction fails.

The directed graph admits a representation of equilibrium as an equation.

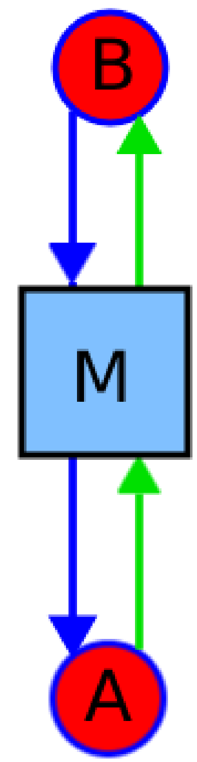


A Market with Money

Reintroducing the market and coloring the graph, blue links represent movement of goods and green the movement of money.

Of course the values must balance.

From the directions of the arrows, infer that A is a buyer and B a seller. There is no need to define *buyer* and *seller* trader types.



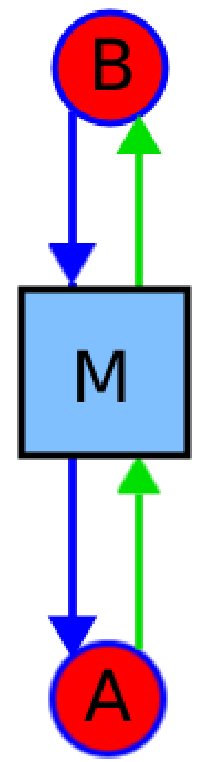
Matching Markets

In *matching markets*, the market develops *because* there are strong complementarities among types.

Agents, not links, are "colored," according to type. Often "color" is represented by position on the right or left of the graph.

The links are directed, representing preferences, not goods flows.

This graph is *two-sided*, meaning both men and women have preferences about partners. The housing market is an example of a *one-sided* matching market.



Using Graphs

- Graphs are helpful in understanding the relationships among actors (like buyers and sellers) and institutions (like markets).
- They make clear what equations need to be defined and solved.
- Institutions may contain more detailed graphs. For example, in the stock market not only investors and issuing firms participate, but also market makers, who keep inventories and smooth out variations in supply and demand.

Network Industries

This section loosely follows Shy, The Economics of Network Industries.

- A *network industry* is one which maintains connections among its clients.
  - A market can be thought of as such a service in pure form, allowing its members to compare prices and arrange trades.
  - Most networks are impure, providing connection plus other services.
- Transportation and communication services may be used or not, along with the conceptual connection.
- A software application's file format may be used by a lone user purely to store information, as well as permitting file sharing among users of the same software.
  - Any standard, whether "official" or simply popular, has the same effect of creating a network.
- Networks create markets.

Network Externalities vs. IRTS in Production

- IRTS in production implies that a single large producer is most efficient, by definition. However, with network externalities in consumption, it is both theoretically possible and seen in practice that several providers share a single network.

- A fixed cost with constant marginal cost implies unbounded increasing returns. The model that leads to Metcalfe's law is far less plausible.

Metcalfe's Law

- To the extent that a network merely *provides connections* between users, its value to each user i depends on the set of connections available. We simplify by assuming that it is not the particular set, but rather the size of the set that matters.

- The simplest estimate of the *value of the network* assumes
  - users are symmetric: $U_i(N) = U(N)$
  - users do not discriminate: $U(N) = u(n)$, where $n = |N|$
  - values are additive: $V = \sum_{i \in N} u(N) = nu(n)$
  - individual value is linear: $u(n) = vn$

  If u_i is nonconstant, we say *network externalities* are present. The linear form $u_i(n) = v_i n_i$ provides a very strong network externality.

- *Metcalfe's Law* is immediate:

$$V = vn^2.$$

A Simple Model with a Network Externality

- We assume a potential market of users M, with $|M| = m$.
- The network externality follows Metcalfe's Law:

$$V = n(nv - c),$$

  where V is the total surplus of the industry, n is the number of users connected to the network, v is the value per connection to each user, and there is a cost of c to stay connected to the network.

  – Unlike the usual theory of the firm, there is a dramatic difference between $c = 0$ and $c > 0$ cases.

- The externality is represented by the coefficient n on v (inside the parentheses).

The Initial Coordination Problem

- Consider the inequality

$$u(n) - c = vn - c < 0,$$

which is the condition where a potential user does not want to join the network.

- It's easy to solve for n:

$$n < \frac{c}{v}.$$

- When $c > 0$ and $v > 0$ is small enough, there may be sizeable populations $n > 0$ such that $u(n) - c < 0$, so the market may fail unless at least $\frac{c}{v}$ users can be convinced to join at the beginning.

- If the initial size of the network is at least $\frac{c}{v}$, the dynamics of the network are qualitatively similar for $c > 0$ and $c = 0$.

Industry Dynamics with Network Externalities

- More interesting than the *fact* that there are increasing returns to size of the market on the demand side is the *effect* of these returns on the dynamics of the industry.

- For example, many innovations start with a single inventor, and as others realize that the innovation is useful, it propagates (or diffuses) through the industry (or even the economy as a whole).

  But with a *pure network good* (one which only offers value by connecting to others) there may be a *minimum viable scale* below which the cost of production is not balanced by the value, even though a large network might have very high net value to each user.

- This means that starting the network requires *coordination* (enough users joining the network at introduction), and therefore the normal market mechanism can fail to support the innovation.

Diffusion Dynamics for a Network Good

- We model the dynamics as a differential equation. The *hazard rate* for joining the network is proportional to the net value to the new user: $\alpha(vn - c)$.
- With m the total population of potential users, multiplying by the *non*user population $m - n$ gives the rate of diffusion:

$$\frac{dn}{dt} = \alpha(vn - c)(m - n),$$

which has the solution

$$n(t) = \frac{m - \frac{c}{v}}{1 + e^{-\alpha v(m - \frac{c}{v})(t - t_0)}}.$$

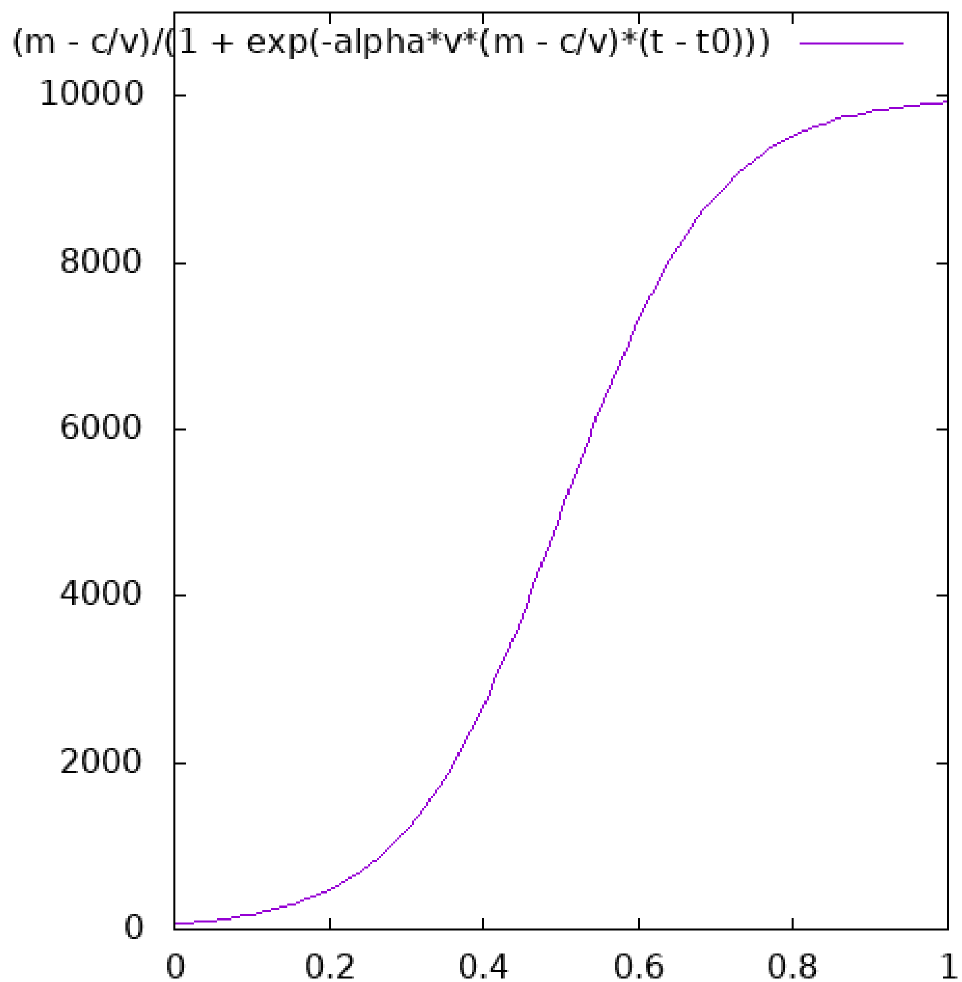
- In the special case of $c = 0$, we can rearrange to get

$$\frac{dn}{dt} = (\alpha v)n(m - n),$$

which is the familar logistic model with solution

$$n(t) = \frac{m}{1 - e^{-\alpha m v t}}.$$

The Logistic Growth Path



The S-shaped logistic growth path is bounded above and below.

Dynamic Competition between Incompatible Networks

- We consider the duopoly, but the principle applies to industries with more than two firms. We have a total population of potential users of m.
- Let the per user per connection values be $v_1 = v_2 = v$, the cost per connection be $p_1 = p_2 = c$, and the number of users (connections) for the two firms be n_1 and n_2.
  - The notations v_i and p_i ("p" for "price") indicate that in a more sophisticated model these might be differentiated or even strategic variables (especially p_i).
- *Incompatible* means that users on one network are *not* connected to the other. Thus for each user, the value of their network is $u_0 = 0$ if not connected to either, and $u_i(n_i) = v_i n_i - p_i$ if connected to network i.

Adoption Decisions of Users

- We suppose that the diffusion of non-users into each network is proportional to net value as in the monopoly case: $\alpha_i(v_i n_i - p_i)$. Once again we will assume symmetry: $\alpha_1 = \alpha_2 = \alpha$.
  - This assumption is more plausible than the assumptions for the "strategic" variables.
- We assume no switching cost, and that existing users switch from 2 to 1 according to the difference in net values: $\delta((v_1 n_1 - p_1) - (v_2 n_2 - p_2))$.
  - Note this hazard rate may be negative.
  - If you were wondering why the hazard rates for non-users have the same α, this switching can help justify that assumption.
  - In a course in economic dynamics, you'd be asked to show when the model with $\alpha_1 = \alpha_2$ and a high δ is equivalent to $\alpha_1 \neq \alpha_2$ and a lower δ.

The Diffusion Model

- Make all symmetry assumptions, and $n_1 > n_2$ at the start of time.
- Then we have

$$\begin{aligned}\dot{n}_1 &= \alpha(vn_1 - c)(m - n_1 - n_2) + \delta v(n_1 - n_2)n_2 \\ \dot{n}_2 &= \alpha(vn_2 - c)(m - n_1 - n_2) - \delta v(n_1 - n_2)n_2\end{aligned}$$

  Conceptually there are also terms $\pm\delta v \max\{n_2 - n_1, 0\}n_1$ in each differential equation, but on the assumption $n_1 > n_2$, they are zero. On that assumption, we can omit the max in the equations above.

- It is easy to see that $\dot{n}_1 > \dot{n}_2$, and for small enough c, $\dot{n}_1 > 0$. (The last is non-trivial to prove because in the limit non-users and n_2 go to 0.)
- Thus $\frac{d}{dt}(n_1 - n_2) > 0$. $\frac{d}{dt}(m - n_1 - n_2) < 0$ if $\dot{n}_2 \geq 0$, so eventually $\dot{n}_2 < 0$.
- Even with $\dot{n}_2 < 0$, $|\dot{n}_1| > |\dot{n}_2|$, so $\frac{d}{dt}(m - n_1 - n_2) < 0$. $n_2 \to 0$ and $n_1 \to m$.
- Symmetry implies that the opposite conclusions hold if $n_2 > n_1$, so this model is "tippy": whichever network starts out ahead soon crowds out the other.

Dynamic Games

- Mathematical analysis of even the simplest game is quite complex. It's easy to see that if the symmetric model is extended so that each firm can choose price p_i, the firm that starts with greater n_i has a big advantage.
  - As long as that firm is willing to match $p_i = p_j$, it will win the whole market.
  - If the monopoly is expected to continue for a long time, firms may even be willing to offer negative prices.
- If everything is symmetric, the game is very similar to the "War of Attrition", which is known to have only mixed strategy equilibria.

Compatible Networks

- As mentioned, for many networks an *interconnection standard* can be created. This means that (subject to quality of service considerations for the "foreign" users) the network externality is based on the sum of users of all networks in the "internet."
  - Large networks don't have a competitive advantage: several networks of different sizes can share the market.
  - Market structure (number of companies) is more stable.
  - The value to each user is greater (approximately double in the duopoly) so price increase may be more than enough to compensate the leader for allowing interconnection.
- Examples: "The" Internet, protocols such as the "World Wide Web," standards like the "DOM" for web browsers (allows Javascript to work on different browsers) and "ODF" for office automation
- Competition is based on price and service quality.

Standards and “Open Source”

- “Open” standards (no royalty to implement) lead to “open source” implementations.
  - “Poor” or hobbyist programmers write their own implementations and contribute them.
  - Business customers trying to avoid “lock-in” may write their own implementations and contribute them when they are not mission-critical or competitive advantage.
  - Open source businesses may implement to support a further value-added product or service.

Homework 2

1. Give two examples of networks you participate in or use. For each example, describe the network as a graph:
   (a) What are the objects?
   (b) What are the links?
   (c) Is the graph directed? Explain why or why not.
   (d) Is it colored? What do the "colors" represent.
   (e) Is it a multigraph? Explain why or why not.
   (f) Do the objects have important attributes (other than their links)? What are they?

   For full credit, use one example nobody else in the class uses. For each additional person who uses your less unique example, the point score will be multiplied by 0.9.

Homework 3

Mathematicians, like game theorists, like to give their examples cute names that are memorable. Explain the following names (or labels) for graphs shown in Lecture 1.

1. "Envy" (slide 23). (This was originally called "Jealousy," but "Envy" is more accurate in English.
2. "Infidelity" (slide 24).

Homework 4

Suppose that each user i has a random value of being connected to each other user j, $u_i(j)$, and the values are independently and identically distributed. Suppose that users are added to the network in a random sequence.

1. Does Metcalfe's law hold?
2. If so, prove it ...
3. ... and suggest generalizations where it still holds.
4. If not, give a counterexample ...
5. ... and suggest restrictions that might make it hold.