# Economics of Information Networks

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**Abstract**

Introduction to economics and networks, the course, the instructor, and the field.

# Everything I need to know I learned in *Economics of Information Networks*

- This is one of the most important courses you will take in the Service Engineering program.
  - That's really just a joke.
  - I just like to say that because it *was* true of my *Basic Data Analysis* course.
- Of course, all the professors say that their course is really useful and interesting ... what's special about *this* course?
- In other words, "Is this course really going to be useful to me?"

# I'm glad you asked!

- Information networks are a crucial aspect of economic progress.
- Of course, today the Internet generates a lot of value-added directly, through games and search engines, cloud services and improved efficiency of communication.
- But in fact, all social activities are mediated by networks: communication networks and transportation networks, networks of friends, networks of allies, networks of business contacts.
  - The old saying, "It's not *what* you know, it's *who* you know" (that leads to success in business and society) is a direct reflection of this fact.
- Network effects are an important cause of *agglomeration economies* (which is a fancy way of saying "people and businesses benefit by gathering in cities").

# I'm *really* glad you asked!

- In economics, we started by taking a very abstract view. A network is simply a collection of economic agents (households, businesses, and governments) that can trade some good with each other.
- In many markets, simply increasing network size makes the network more valuable.
  - This isn't true for typical private goods: your costs don't decrease unless you actually sell more, and the pleasure of consumption is not dependent on how many other people consume the same good.
  - But for goods like telephones (the more of your friends have one, the happier you are), it is true.
- This simple fact has important implications for network growth and competition in that market.

# I'm *still* glad you asked!

- By looking at network *structure*, we can learn a lot about costs of providing a distributed service such as communication and transportation.
- Network structure also has important implications for bargaining power (if everybody knows the same people you know, they don't need to ask for an introduction, and won't owe you any favors) and employment opportunities (if somebody you know knows somebody who has a job you can do, they can connect you).
- Network structure can help to predict alliances in politics and business.
- Network structure can help you avoid the high cost and low efficiency of broadcast media if you can identify a network that connects the people you want to reach.

# Objects of analysis of networks

- Simple, abstract network externalities without concern for structure. We just count connected members. Analysis is done via the usual microeconomic foundations of utility functions, cost functions, and profit functions, plus markets.
- Network flows, usually in fixed networks such as communications and transportation networks. This is a specialized area of operations research which is highly developed.

  However, in modern electronic information networks, flow constraints are usually rather secondary, so we will treat this subject only briefly and leave a full discussion to specialized courses.
- Network structure, both fixed and endogenous. Network structure is modeled using *graph theory*.

# Brief course description

**Goal** Understanding of the basic ideas of network analysis using economics, operations research, and graph theory.

**Overview of the Lectures** We consider basic ideas about modeling networks as graphs. Then we introduce simple economic models, followed by network flow analysis. In the second part (the majority) of the course, we apply these models to various kinds of information networks, including communications networks and the Internet. We will also discuss the nature of *security* in a network environment.

# Prerequisites and Language

**Prerequisites** Although not absolutely necessary, for best results students should have taken college level calculus and linear algebra courses, and an introductory microeconomics course.

**Language of Instruction** I plan to lecture in English, and original course materials will generally be in English. I will accept and answer questions in Japanese to the extent possible (but my technical vocabulary is relatively weak; it's probably best to use English technical terms where possible).

- Someday, I hope to provide Japanese translations of some course materials. Probably not this time however, because I am extremely busy just keeping up with preparations for my courses and other work.

# Homework: Manual Calculation

- Calculation by hand will be a prominent feature of this class. N.B. "By hand" includes use of spreadsheets, but unfortunately I can't permit that on examinations.
- Intended to improve your understanding and intuition about computations.
- Computers can do calculations more quickly, more accurately, and at far larger scale than any human is capable of, but they are a black box to any but expert software engineers. The "garbage in, garbage out" problem is perhaps the most dangerous fallacy in business research.

# Homework: Computational Exercises

- Computational exercises *may* be assigned.
- Intended to help you discover the structure of some information networks.
- The intent is not to make you an expert at using computers.

# Resources: URLS

- Just about anything you need to know about the class will be on the class home page, `http://turnbull.sk.tsukuba.ac.jp/Teach/EconInfoNet/`. If it's posted on the class home page, "I didn't know (about the assignment, test, *etc.*)" will *not* be an acceptable excuse.
- The other important URL is my personal calendar, `http://turnbull.sk.tsukuba.ac.jp/schedule.html`.

# Resources: Textbooks

- David Easley and Jon Kleinberg, *Networks, Crowds, and Markets* is *required* because it is available online (see home page for link). I recommend you buy it anyway if you have any interest in the technical analysis of information networks – this is a classic. Exercises will be assigned from this book.
- Oz Shy, *The Economics of Network Industries*. Exercises will be assigned from this book.

See the class home page for more references.

# Introduction: Why Study Networks?

*Please read Easley and Kleinberg,* Networks, Crowds, and Markets*, Ch. 1. This section is complementary to that chapter – it is not the same!*

The word "network" has become widely used to describe "connectedness":

- Electronic data transmission networks such as *The Internet*, organizational *intranets*, *local area networks* (LANs), *virtual networks* carried over the public Internet, *etc.*
- Physical networks for communication and transportation
- Social networks mediated by websites such as Facebook, Mixi, and Twitter
- Networks of companies, such as a manufacturer's suppliers (*keiretsu*) and cooperative *corporate groups*
- *P2P* networks for file and network load sharing
- Networks of personal contacts (and maintenance and extension of such contacts is called "networking")
- Even users of the same wordprocessing software are connected by their ability to share documents

# What is a network?

- A *network* is a *collection of entities connected to each other*. A group of related examples include:
  1. A group of computers connected to another by Ethernet.
  2. The users of the computers in (1) connected by email (a different network, especially if the computers are in a shared university lab).
  3. The users of the computers in (1) connected by Google chat (a different network from either (1) or (2)).
  4. The authority relationships in a company.
  5. A group of people connected by friendship.
  6. A group of Facebook pages connected by “friend” tags (a different network from (5)).
- And something very different is a bunch of railroad stations connected by track.

# Networks, links, and graphs

- It is often convenient to represent a network in terms of bilateral *links*, *i.e.*, a relation between specific pair of entities (usually called *objects* or *nodes*, and in math, *vertices*).
- Many of the connections in the examples above decompose easily into links.
  - *E.g.*, in the group of friends, each pair of people is friendly with the other. And if some pair of friends should fight, that doesn't dissolve the group: the other pairs are still friends.
  - Even though one email user can address a single message to several others, we can think of this as a set of pairwise links, each containing the author and one of the recipients.
- A collection of *objects* and *links* is called a *graph*.

# Graphs

- A *graph* is a *collection of objects and links.* Each link connects exactly two objects (which can be the same: this is called a *loop*).
- There are many kinds of graph, depending on the exact nature of links.
- In some graphs, links are symmetric: if object $A$ is linked to object $B$, then object $B$ is linked (also, *connected*) to object $A$. These are *undirected graphs*.
- In other graphs, links are asymmetric. There can be a *link from $A$ to $B$*, but no link from $B$ to $A$. These graphs are called *directed graphs*. In directed graphs links are often called *arrows*.
  - A directed graph can represent an undirected graph by *imposing* symmetry: for every link from $A$ to $B$, there must be a link from $B$ to $A$.
  - In discussing directed graphs, the meaning of *connected* may be surprising: $A$ and $B$ are *connected* if there is a link from $A$ to $B$ *or* there is a link from $B$ to $A$ (or both, but one or the other may be missing). This means that in $A \to C \leftarrow B$, $A$ and $B$ are connected even though there is no way to get from $A$ to $B$

# Multigraphs

- In many of the networks we discuss, it makes little sense to have multiple links between two nodes. What does it mean to have *two* (different) friendship relations with a person?
- In other cases, there may be multiple links (a long web page may link to another web page twice). A graph in which multiple links are allowed is often called a *multigraph*.
- Multigraphs may be directed or undirected; if it's important to distinguish them, we write "directed multigraph" and "undirected multigraph".
- An important kind of directed multigraph in mathematics is a *category*. A category has two special properties: it is *reflexive* (there is always a special link from each object to itself, called the *identity* link), and it is *transitive*, such that if there is a link from A to B and a link from B to C, there must also be a link from A to C.
  - Categories arise naturally when considering *paths* (sequences of links) through a graph.

# Abbreviations

- As usual, we abbreviate terms when the specific definition is clear from context.
- Specifically, *directed*, *undirected*, and *multi*- will be omitted frequently.
- The type of graph in examples will often be clear from the diagrams.

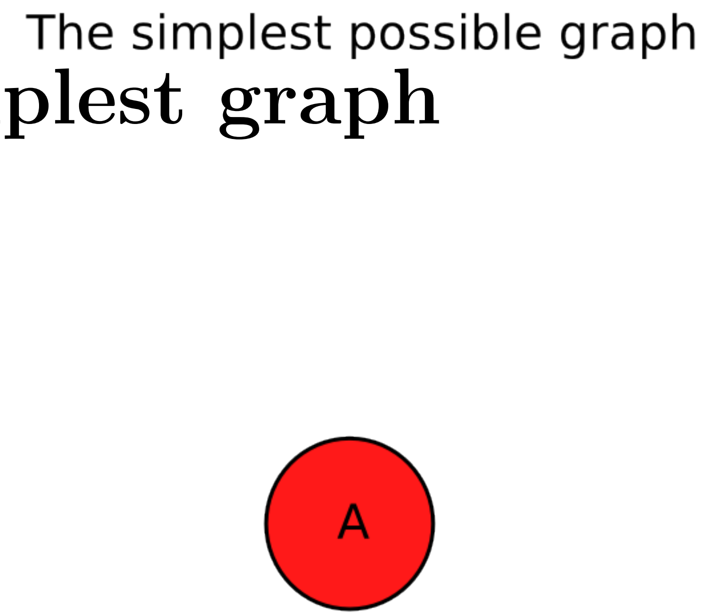
# Link attributes

- Links may have *attributes* or properties. In communications and transportation, *capacity* is very important. A link with low capacity becomes a *bottleneck*, impeding flow, if there are high-capacity links connected to it. Graphs with links possessing a quantitative attribute are called *weighted graphs*.

- It may be useful to distinguish *types* of links. In a multigraph, a group of Internet users may communicate by email, by Twitter, and by posting to their Facebook pages. $A$ and $B$ may be linked by email and Twitter, while $B$ and $C$ are linked by $C$'s Facebook page where $B$ makes comments and by email. Graphs with different types of links are called *colored graphs*.
  - A *directed graph* can be represented by an *undirected colored graph* and an order on the objects, so that an "up" link goes from the lower to the higher, and a "down" link goes from the higher to the lower.

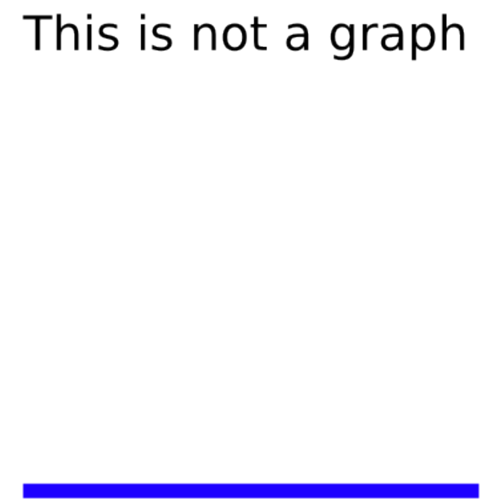
- Objects can have attributes, rarely as interesting as link attributes. (*Tokens*, which are "moving attributes" of objects, are a basic component of Petri net theory, used in the theory of *parallel computing*.)

# The simplest graph



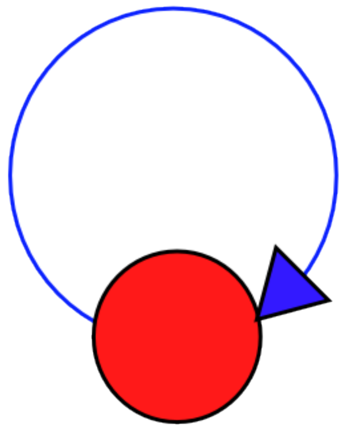

A single node with no links.

# Not a graph




Each link in a graph must connect two nodes (or possibly a single node with itself).

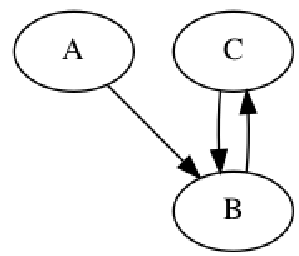
# The simplest interesting graph



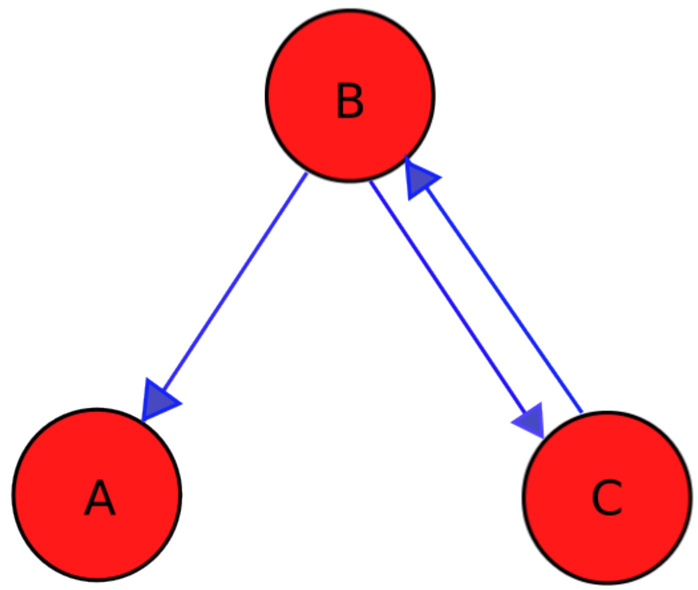
A directed graph with a single node and a single link.

# A graph with symmetric and asymmetric links

Why is this graph called "Envy"?
(Previously it was called "Jealousy".)



# Another graph with symmetric and asymmetric links




Why is this graph called "infidelity"?

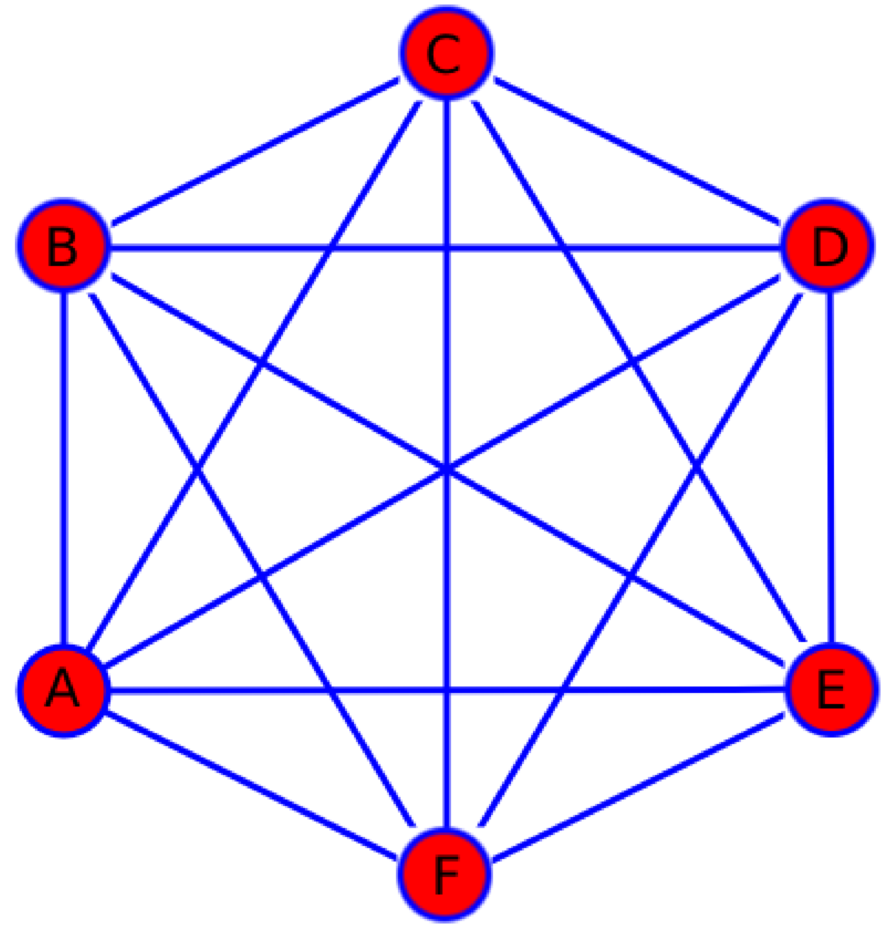
# Graphs of real networks

- Following Ch. 1 of Easley and Kleinberg.
- Some of these networks are very personal, but some have global impact.

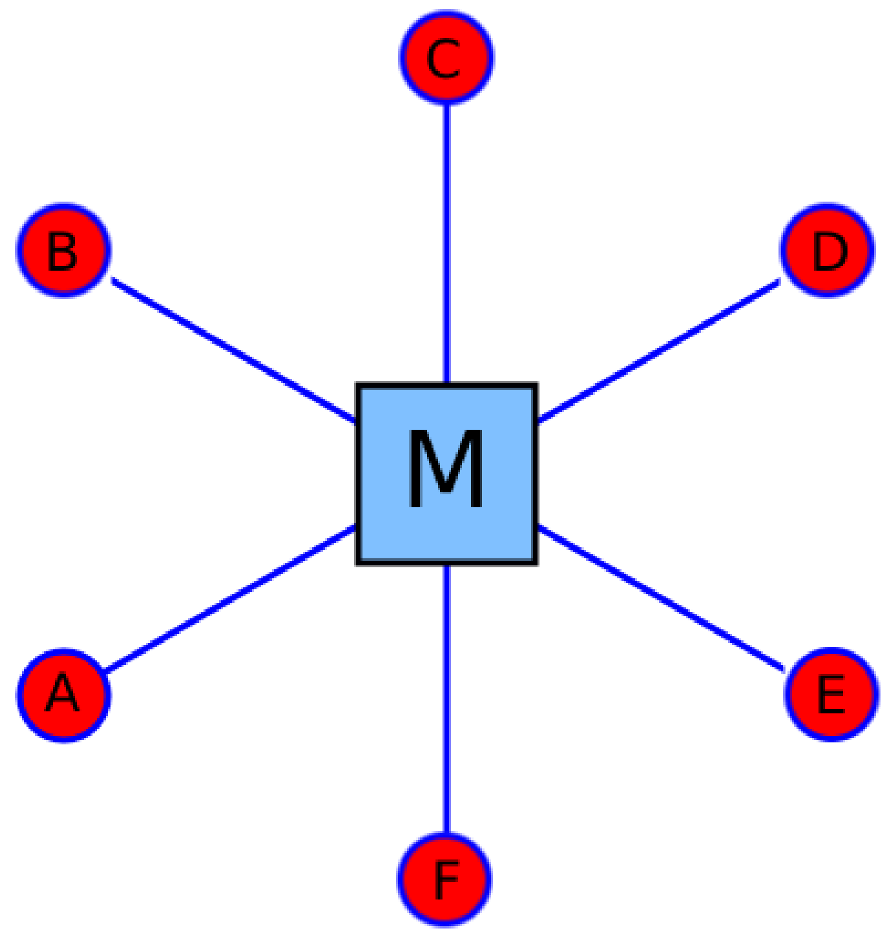
# Network Structure of Markets

The simplest network model of a market simply connects traders to each other.

- No distinction between buyers and sellers, or between buying and selling.
- All traders are "in the same place"—as a network, all connected to each other.



# An Alternative Model of the Market



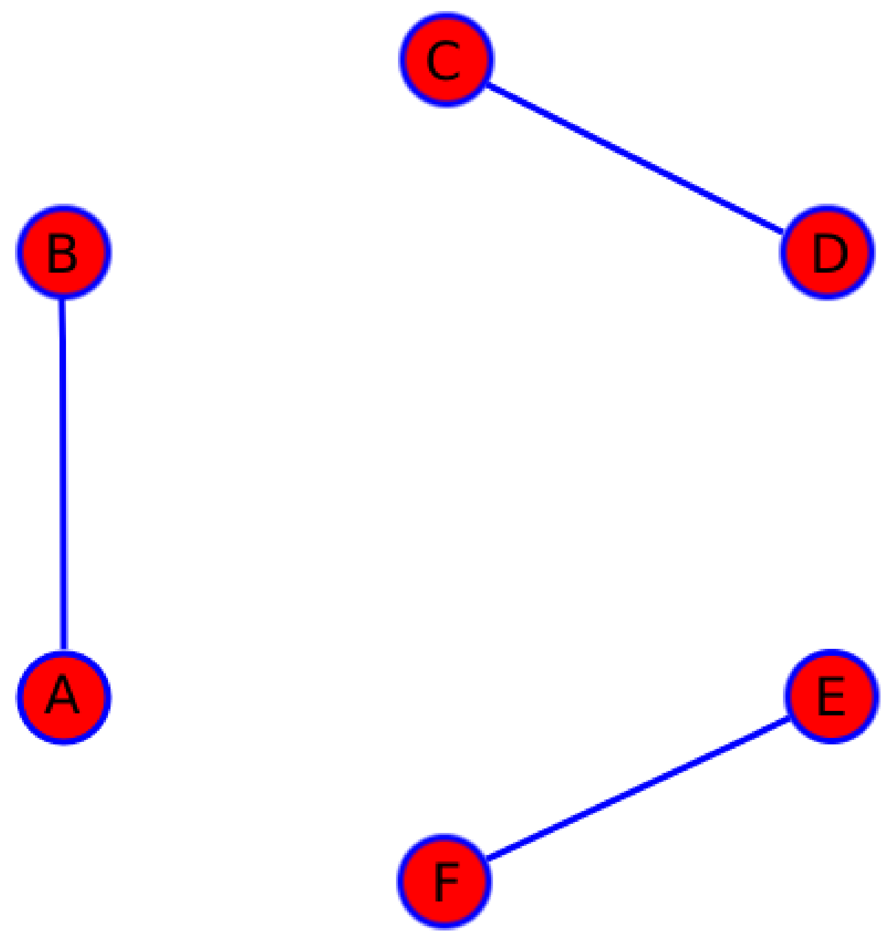

Another simple model connects the traders *through* the market.

# The Logic of the Star Market

The links in the two market models are *symmetric*. It's easier to see what this means if we consider a much simpler structure: a set of *barter* relationships.

It's clear that something is missing from the symmetric link: it doesn't provide a way to talk about *equilibrium*, that is, the balance of value given for value received. Symmetry *assumes* balance.

The star arrangement allows the market to assure that balance, and achieve efficiency. Barter won't be efficient because *each* individual trade must balance.

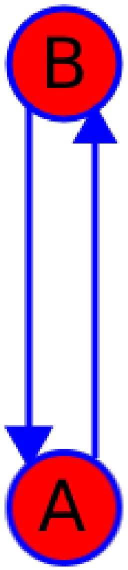


# A Directed Graph Approach

Consider one pair in isolation, with directed links indicating transfer of a value from one to the other.

If one link were missing, then balance could not be achieved and the transaction fails.

The directed graph admits a representation of equilibrium as an equation.

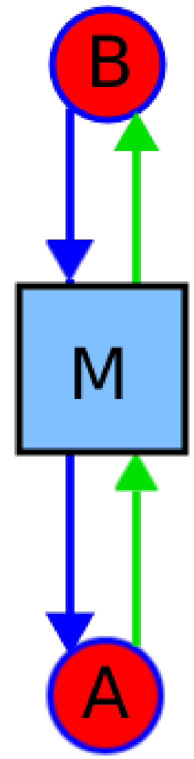


# A Market with Money

Reintroducing the market and coloring the graph, blue links represent movement of goods and green the movement of money.

Of course the values must balance.

From the directions of the arrows, infer that A is a buyer and B a seller. There is no need to define *buyer* and *seller* trader types.



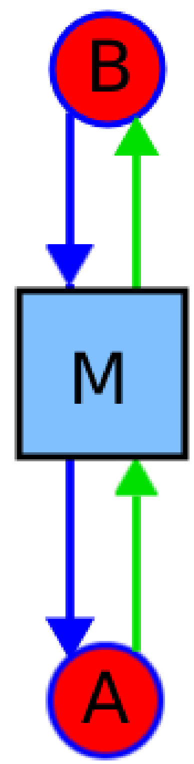
# Matching Markets

In *matching markets*, the market develops *because* there are strong complementarities among types.

Agents, not links, are "colored," according to type. Often "color" is represented by position on the right or left of the graph.

The links are directed, representing preferences, not goods flows.

This graph is *two-sided*, meaning both men and women have preferences about partners. The housing market is an example of a *one-sided* matching market.



# Using Graphs

- Graphs are helpful in understanding the relationships among actors (like buyers and sellers) and institutions (like markets).
- They make clear what equations need to be defined and solved.
- Institutions may contain more detailed graphs. For example, in the stock market not only investors and issuing firms participate, but also market makers, who keep inventories and smooth out variations in supply and demand.

# Network Industries

*This section loosely follows Shy,* The Economics of Network Industries*.*

- A *network industry* is one which maintains connections among its clients.
  - A market can be thought of as such a service in pure form, allowing its members to compare prices and arrange trades.
  - Most networks are impure, providing connection plus other services.
- Transportation and communication services may be used or not, along with the conceptual connection.
- A software application's file format may be used by a lone user purely to store information, as well as permitting file sharing among users of the same software.
  - Any standard, whether "official" or simply popular, has the same effect of creating a network.
- Networks create markets.

# Network Externalities vs. IRTS in Production

- IRTS in production implies that a single large producer is most efficient, by definition. However, with network externalities in consumption, it is both theoretically possible and seen in practice that several providers share a single network.

- A fixed cost with constant marginal cost implies unbounded increasing returns. The model that leads to Metcalfe's law is far less plausible.

# Metcalfe's Law

- To the extent that a network merely *provides connections* between users, its value to each user $i$ depends on the set of connections available. We simplify by assuming that it is not the particular set, but rather the size of the set that matters.

- The simplest estimate of the *value of the network* assumes
  - users are symmetric: $U_i(N) = U(N)$
  - users do not discriminate: $U(N) = u(n)$, where $n = |N|$
  - values are additive: $V = \sum_{i \in N} u(N) = nu(n)$
  - individual value is linear: $u(n) = vn$

  If $u_i$ is nonconstant, we say *network externalities* are present. The linear form $u_i(n) = v_i n_i$ provides a very strong network externality.

- *Metcalfe's Law* is immediate:

$$V = vn^2.$$

# A Simple Model with a Network Externality

- We assume a potential market of users $M$, with $|M| = m$.
- The network externality follows Metcalfe's Law:

$$V = n(nv - c),$$

  where $V$ is the total surplus of the industry, $n$ is the number of users connected to the network, $v$ is the value per connection to each user, and there is a cost of $c$ to stay connected to the network.

  - Unlike the usual theory of the firm, there is a dramatic difference between $c = 0$ and $c > 0$ cases.

- The externality is represented by the coefficient $n$ on $v$ (inside the parentheses).

# The Initial Coordination Problem

- Consider the inequality

  $$u(n) - c = vn - c < 0,$$

  which is the condition where a potential user does not want to join the network.

- It's easy to solve for $n$:

  $$n < \frac{c}{v}.$$

- When $c > 0$ and $v > 0$ is small enough, there may be sizeable populations $n > 0$ such that $u(n) - c < 0$, so the market may fail unless at least $\frac{c}{v}$ users can be convinced to join at the beginning.

- If the initial size of the network is at least $\frac{c}{v}$, the dynamics of the network are qualitatively similar for $c > 0$ and $c = 0$.

# Homework Submission

1. Submit your homework *by email to*

   `"Economics of Information Networks" <turnbull@sk.tsukuba.ac.jp>`

   The `Subject:` should be `01CN901Homework #1`. (For assignments #2, #3, and so on, adjust the homework number.)

2. Without the class number and the homework assignment in hankaku romaji, your email may get lost due to spam filtering. Use the class number above, even if you are registered according to a different code.

3. Your email must contain your *name* and *student ID number*.

4. For simple answers, I *strongly* prefer *plain text* or TEX notation for expressions and equations to *Word documents* and *HTML*. In plain text, you may write subscripts using functional or programming notation (*i.e.*, $X_t$ becomes `X(t)` or `X[t]`), and superscripts using the caret (*i.e.*, $X^t$ becomes `X^t`) or double-star ($X^t$ becomes `X**t`).

# Homework Hints

Many of the tasks assigned in homework are expressed using idioms specific to this class. A few of these words are mentioned below, along with the specific requirements they indicate.

**solve** Also **give a solution** or **derive**. You *must* show your work. Obvious calculations of common operations, such as the $6 \times 5 \times 4 \times 3 \times 2 \times 1$ in $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ may be omitted, but even slightly more complex operations such as ${}_6C_3 = \frac{6!}{3!3!} = 20$ should be written out. Fractions should be in lowest terms, but do not need to be reduced to decimals. Square roots of perfect squares should be reduced if you recognize them, but all roots *may* be left in the standard notations $\sqrt{x}$ or $x^{\frac{1}{2}}$ or similar as seems most appropriate.

**discuss** Most important, relate the computation to the real problem in economics (or physics or biology for some of the "toy" examples). Especially mention anything paradoxical, surprising, or extreme about the interpretation of the result in context of the real problem.

**compare** Like **discuss**, but more specific: you should use statements of the form "*this* is the same as *that*," "*this* is different from *that*," and (best) "*this* is similar to *that*, except ...."

When appropriate use quantitative or ordering comparisions: more/yes, sooner/later, *etc.*

**show *expr* is *expr*** Often you need to transform one of the expressions to the other. You must show your work, not just "expr $a =$ expr $b$ (same!)"

**notation** You may define your own notation. For example, in Q#2 you may be asked to compare $\delta$ in Q#1 to $\delta$ in Q#2. This gets confusing and long winded (*i.e.*, because you write "$\delta$ of Problem 1" over and over again). It may be useful to rewrite one of the results by substituting $\gamma$ for $\delta$ everywhere.

# Homework 1

- Get Easley and Kleinberg [2010]. Download the current PDF version, or bookmark the HTML version.

  For proof, just send me an email saying you did so.

# Homework 2

1. Give two examples of networks you participate in or use. For each example, describe the network as a graph:
   (a) What are the objects?
   (b) What are the links?
   (c) Is the graph directed? Explain why or why not.
   (d) Is it colored? What do the "colors" represent.
   (e) Is it a multigraph? Explain why or why not.
   (f) Do the objects have important attributes (other than their links)? What are they?

   For full credit, use one example nobody else in the class uses. For each additional person who uses your less unique example, the point score will be multiplied by 0.9.

# Homework 3

Mathematicians, like game theorists, like to give their examples cute names that are memorable. Explain the following names (or labels) for graphs shown in Lecture 1.

1. "Envy" (slide 23). (This was originally called "Jealousy," but "Envy" is more accurate in English.
2. "Infidelity" (slide 24).

# Homework 4

Suppose that each user $i$ has a random value of being connected to each other user $j$, $u_i(j)$, and the values are independently and identically distributed. Suppose that users are added to the network in a random sequence.

1. Does Metcalfe's law hold?

2. If so, prove it …

3. … and suggest generalizations where it still holds.

4. If not, give a counterexample …

5. … and suggest restrictions that might make it hold.