情報ネットワークの経済学
Economics of Information Networks

最終試験- Final Examination

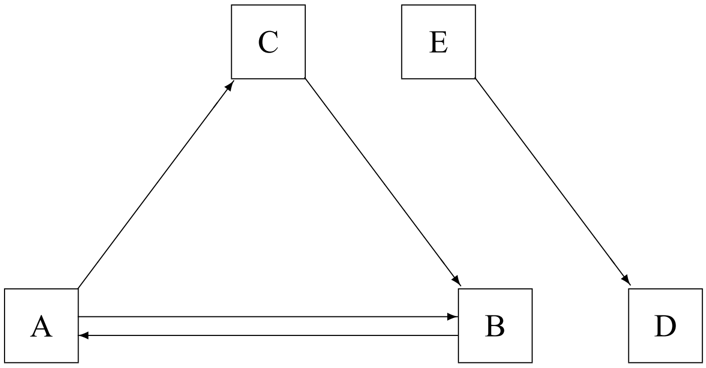
平成２８年１２月１５日- December 15, 2016

Problems

When you are asked to do a calculation, you do not need to compute the decimal equivalent of a fraction, or radical (square root). Fractions should be reduced to lowest terms for convenience in grading. Radicals do not need to be reduced. That is $\frac{7}{5}$ and $1\frac{2}{5}$ are OK, please try to avoid $\frac{28}{20}$. $\sqrt{12}$ is OK.

1. `[Problem ID #1] Interpretation of directed networks`

Consider this graph representing a network.



(a) Explain what the pieces of the graph mean, including the labelled squares, the connecting lines, and the arrowheads.

The squares are called nodes*, and they are objects which may be connected. In social and economics networks, nodes are agents (decision-makers). The lines are* links *or* edges *(in the directed case often called* arrows*). They indicate some kind of relationship. The arrowheads indicate that the relationship need not be symmetric. The relationship of A to B is symmetric, since B is also linked to A. The relationship of A to C is asymmetric. C is not linked to A. The relationship of E to D is like that of A to C. A, B, and C are unrelated to D and E.*

(b) Consider a network of people who communicate by email. Explain how the pictured network might arise naturally. (Hint: this state is probably temporary.)

The simplest explanation is that A has sent at least one email to B and C, C has sent email to B but not A, and B has sent email to A but not C. This kind of situation might arise naturally if C is a new member of an organization, or C is a customer, B is the customer service representative, and A is an engineer. In the second case A and B have a long-term relationship, but C only needs one simple explanation. Of course C probably sends a mail to A to thank her, adding a new link from C to A. The relationship of E to D is like that of A to C. A, B, and C are unrelated to D and E.

(c) What are the *connected components* of this graph?

{A, B, C} and {D, E}.

(d) Is this graph connected? Give the reason for your answer.

No. By definition a connected graph has a single component, but this graph has two. Also, there is no (undirected) path from A to D.

2. `[Problem ID #2] Metcalfe's Law`

   Consider Metcalfe's Law.

   (a) Give an expression for the value of a network by Metcalfe's Law.
   $V = cn^2$, where V is value, c is a constant representing value per link, and n is the number of nodes.

   (b) Explain the assumptions used to derive Metcalfe's Law.
   There are many ways to derive Metcalfe's Law. Here's one (from class):

   - *users are symmetric in the way they value: $u_i(N) = u(N)$*
   - *users do not discriminate among connections: $u(N) = u(n)$, where $n = |N|$*
   - *values are additive: $V = \sum_{i \in N} u_i(N) = nu(n)$*
   - *individual value is linear: $u(N) = cn$*

   where N is the set of users, and u_i is agent i's utility function for a network N (which is a graph of nodes and links).

   (c) Suppose a network satisfies the assumptions for Metcalfe's law, except that each person in the network has 100 friends and family they want to connect to, and each connection has value $v > 0$. Give an expression for the maximum possible value of this network for each network N with $|N| > 100$.
   For $n > 100$, the "best" possible case for the users is that all their friends are in the network. Then each person's value is $100v$, and the network's total value is $100vn$.

3. `[Problem ID #5] social network: clustering`
   Consider the evolution of a social network.

   (a) Define *triadic closure*.
   Triadic closure is a process in social networks that occurs when A is connected to B and to C, but B is not connected to C. "Closure" occurs when B and C become connected in the future.

   (b) Define the *clustering coefficient* of a node.
   The clustering coefficient of a node x is

   $$\frac{2 \sum_{i \in N(x)} |C(i)|}{|N(x)|(|N(x)| - 1)},$$

   where $N(x)$ is the set of neighbors of x and $C(i)$ is the set of links from i to other members of $N(x)$. (The denominator of the fraction is the number of pairs from $N(x)$, i.e. the maximum possible number of connections.

(c) Explain how *triadic closure* and the *clustering coefficient* are related.

The clustering coefficient quantifies the amount of triadic closure around a node.

4. `[Problem ID #3] personal relation to networks`

(a) Give three different examples of networks you use or participate in. One should be a *physical* network, one should be an *information* network, and one should be a *virtual* network. Be sure to say which example is of each type!

Some examples: physical network = Tokyo subway/rail system; information network = mailing list for M1 students of Shako; virtual network = users of Microsoft Word, who can exchange files.

(b) What is your master thesis research theme?

Varies by student, of course.

(c) Explain in about 75 words how information networks relate to your theme. (Even if you do not directly discuss them in your thesis, they are surely related!)

This answer depends on the theme, of course, but here are some typical examples:

- *There are other people working on related themes, and we are helped by reading others' papers which we access on the WWW.*
- *Information networks like the Internet are an important cost-reducing factor for the industry I am studying.*
- *Information networks like the Internet create the opportunity for new services in the industry I am studying.*
- *Without the Internet my industry couldn't exist.*
- *I am studying the communications and Internet industry.*