名前 _____ ID# _____ 1

Economic Dynamics

SAMPLE Examination Questions and Answers

December 21, 2018

General instructions / 一般説明

Japanese translations may follow the English text. If the English and Japanese versions of any text differ in meaning, the English text is correct. Please ask for clarification if you have any doubt. Any corrections will be posted.

英文に次いで和訳がある場合がある。英文と和文の間に食い違いがあれば、英語の方が正しい。ただし、不明な点については遠慮なく聞いてください。あらゆる訂正は掲示します。

Be sure to write your name and student ID number on each sheet.

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** In Japanese, please take great care in writing kanji. Avoid abbreviated kanji; the only one I know is the 3-stroke mongamae.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited. Some dictionaries may be provided at the examination.

As usual, the only items that should be present on your desk are pencils, pens, erasers, pencil sharpener, watch, and the examination paper. Put other items in your bag and place the bag under the seat or desk, or on the seat next to you.

Except for calculations, I can give complete answers to most problems within 3 lines. Some questions can be answered correctly with 2 or 3 words. Students usually more space to express their answers, but you should try to avoid making these problems harder than I intend them to be. Answers will be evaluated on presence of the correct idea, not the quantity of words used.

Below each problem enough space is provided for a complete answer. Please write your answers there. If you need more space for an answer, use any available empty space. Clearly indicate where the additional text is, and label it clearly with the question it answers. A figure with axes is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used, and if needed, how you derived them.

General instructions / 一般説明

名前と学籍番号を忘れずに各ページに記入してください。

以下に経済動学の知識を検定する問題のすべてに解答せよ。**解答の言語は日本語でも英語でも構わない。**もし日本語で書けば漢字などの書き方に十分注意してください。たとえ、省略した漢字などを使わないこと。（私が読めない場合には省略した文字を「間違え」と採点する。）

ノート・教科書・辞書・電卓・携帯電話・その他のメモリを持つ電子製品の用は禁止である。全ての計算は簡単であるので電卓などは必要ない。

机の上にペン・鉛筆・消しゴム・鉛筆削り・時計・この試験用紙の他の物を置かないこと。その他のものを側の席に置くこと。

計算問題以外の問題には私は３行以内十分に答えられる。少数の問題は６文字でも可能だ。もちろん学生の方がより多くのスペースを要するが、私の意味より難し過ぎることを考えないでください。採点ははっきり正しい概念が含まれているかどうかによって判断し、それ以上に文字の数は構わない。

各問題の下に十分にスペースを用意するのでそこに買いてください。それ以上のスペースが欲しかったらどの空白でも構わない。ただし、用意したスペースに「追加あり」を示し、追加のテキストにどの問題の答えかをはっきり表すこと。グラフ問題には軸を用意するのでそれを使用すること。計算問題には結果だけは少数点（０可）しか与えなく、使用した式などが必要だ。場合により、導き出す方法も表すこと。

Problems / 問題

Each question is worth 10 points, unless otherwise specified.

1. `[Problem ID #1] optimization: durable goods monopoly`
   Explain how a monopolist selling a *durable good* faces "competition with himself."

2. `[Problem ID #2] exhaustible: fishery LR equilibrium, phase diagram`
   In the case of a fishery with X firms (boats), average productivity $f(Z, X)$, positive cost of extraction p, and price for consumption q, we saw that we could use the zero-profit condition for long-run equilibrium to give the equation $p = qf(Z, X)$, and that we could solve to get the long-run number of firms in the industry as $X = M(Z)$.

   (a) Give an expression for the total harvest Y in long-run equilibrium as a function of Z.

   (b) On the graphs below, draw the law of natural increase $H(Z)$ and the equilibrium harvest function $Y(Z)$ from part 2a. On the left graph, show the case with a stable steady state. On the right graph show the case where population collapses to zero due to overharvesting.

3. `[Problem ID #3] exhaustible: renewable; law of increase`
   Let $H(Z)$ be the law of increase for a population Z. Explain the economic meaning of the *own rate of return* $H'(Z) = \lim_{\delta \to 0} \frac{H(Z+\delta) - H(Z)}{\delta}$.

4. `[Problem ID #4] exhaustible: backstop technology, choke price`

   Explain the relationship between a *backstop technology* such as solar power and the possibility of a *choke price* for an exhaustible resource such as oil. Be sure to include definitions of both terms.

5. `[Problem ID #5] exhaustible: and renewable definition`
   Describe exhaustible resources, both the general case of *renewable resources* and the special case of *(pure) exhaustible resources*.
   尽くせる資源について述べよ。一般的な再生可能資源と特定の純粋尽くせる資源を含むこと。

   (a) Give a verbal definition of *renewable resource*.
   言葉で再生可能資源の定義を書け。

   (b) Give a mathematical description (in terms of variables and functions) of a renewable resource.
   数式（変数・関数など）で再生可能資源の定義を書け。

   (c) Give a verbal definition of *pure exhaustible resource*.
   言葉で純粋尽くせる資源の定義を書け。

(d) Give the mathematical restriction that characterizes a *pure exhaustible resource* as a special case of a renewable resource.
再生可能資源の種類として数式（変数・関数など）で純粋尽くせる資源の定義を書け。

6. `[Problem ID #6] exhaustible: price path with extraction cost`

Consider an exhaustible resource with an extraction cost function

$$C(R, S, t) = f(t)Rg(R)G(S)$$

where R is rate of extraction, S is current stock, f is cost decreases from improved technology ($f' < 0$), $g(R)$ is decreasing returns to scale ($g' \geq 0$) and $G(S)$ is increased efficiency of extracting from a larger stock ($G' \leq 0$). The equilibrium condition is

$$\frac{\dot{p}_t}{p_t} = r + \frac{f(t)R_t g(R_t)G'(S_t)}{p_t}$$

. Briefly describe the price path of the resource, comparing it to a resource that can be extracted costlessly, in the short and long run.

7. `[Problem ID #7] exhaustible: rational bubble`
Discuss the possibility of a "rational bubble" in the market for an exhaustible resource.

(a) Describe the price path, the initial price, and the long run behavior of the stock of the resource.

(b) Explain how and why a "bubble" deviates from efficiency.

(c) Explain why market pressure may not restore efficiency.

8. `[Problem ID #8] exhaustible: asset arbitrage, Hotelling Rule`

Following steps (a)–(c), derive the *Hotelling Rule* from the arbitrage condition that the markets for both bonds and the stock of an exhaustible resource be in equilibrium. Also, answer (d).

(a) Formulate the *arbitrage equation* that defines asset market equilibrium in terms of the interest rate, current price of the exhaustible resource, and future price of the exhaustible resource, for a short period of time.

(b) Explain why markets that don't satisfy this condition are not in equilibrium.

(c) Show that by manipulating the equation and taking the limit as the period that the assets are held goes to zero, the Hotelling Rule for the price of the exhaustible resource can be expressed as an equation involving the growth rate of price and the interest rate.

(d) Explain why the Hotelling Rule implies no steady state for the exhaustible resource.

9. `[Problem ID #9] optimization: value function; formula`
   Give a formula defining the *value function* used in backward induction analysis.

10. `[Problem ID #10] optimization: value function; definition, use`
    Suppose the formula defining the function $Z()$ is
    $$Z_t(X_{t-1}) = \max_{s_t} U(s_t(X_{t-1})) + Z_{t+1}(X_t).$$
    What is Z? How is it used in dynamic analysis?

11. `[Problem ID #11] optimization: backward induction: computation`

    Fill in the following table for backward induction. The starting balance is 0, the target balance at the end of three years is 15000. Utility is logarithmic ($u(C) = \ln C$). The discount factor is $\delta = 1$ in all periods.
    You do not need to evaluate the utility function; leave it in the form "$\ln 10000$*" when consumption is 10000, for example. Remember that the logarithmic utility function tries to make consumption equal in each period, if possible.*

12. `[Problem ID #12] optimization: backward induction: tree cutting`

    Consider a company which grows trees and cuts them for sale as lumber. It can only grow one tree at a time. The value of the tree increases according to the function $R(\tau) = R_1\tau^2$. When it cuts the tree, it sells it, then plants another tree of value 0, which starts to grow.

    Assume that the tree can only be harvested at a particular date each year, and that the company is planning for three years. Thus, the tree must be cut at $t = 3$ (and the model ends), and the tree may be cut at $t = 1$ or at $t = 2$, in which case another tree is planted and the problem continues until $t = 3$. The company wishes to maximize the total revenue from selling the tree(s).

    In mathematical terms, let the amount the company receives for selling a tree at time t be S_t. It wishes to maximize $S_1 + S_2 + S_3$. Solve the problem by backward induction.

Year (t)	**1**	**2**	**3**
Income (Y)	20000	20000	20000
Saving ($S = s_t^*(X_{t-1})$)			
Consumption (C)			
Current utility ($u(C)$)			
Ending balance (X)			
Future value ($V_t = V_t^*(X_{t-1})$)			
Optimal saving ($s_t^*(X_{t-1})$)			
Value function ($V_t^*(X_{t-1})$)			

- Let the state variable at time t be A_t, the *age* of the currently growing tree. *E.g.*, if the tree was last cut at time $t = 1$, and "now" is $t = 3$, $A_3 = 2$. Explain why this is a good choice of state variable.

- Compute the optimal revenue for time $t = 3$ as an expression in terms of A_3 (*i.e.*, $S_3 = 0$ if the tree is not cut, or $S_3 = R(A_3)$ if the tree is cut and it is A_3 years old).

- Compute the value function $V_3(A_3)$ as an expression in terms of s_3. [*This should be A_3!*]

- Compute S_1, V_2, and S_2 by maximizing $S_t + V_{t+1}(A_{t+1})$ at times $t = 2$ and then $t = 1$.

- Compute the optimal tree-cutting plan for all three years.

- Describe the meaning of $V_2(A)$ in words.

Note: You may notice that the notation $V_t(A_t)$ (both subscripts are t) is different from that used in class, $V_{t+1}(X_t)$ (one is $t+1$, the other t). This is correct and more convenient for this problem.

13. `[Problem ID #13] dynamics: classifying cases`
A company is growing trees to sell as materials to make wood products. The longer a tree grows, the more wood it contains, and the more revenue it brings. The company must decide when to cut and sell each tree. Is this a dynamic problem? Explain why or why not.

14. `[Problem ID #14] dynamics: classifying cases`
Consider the following two situations involving time. One is dynamic and the other is not.

    - **Case A:** An oil-exporting country decides to try to increase oil production by 10% per year for many years to come.
    - **Case B:** A country that makes computer memory chips decides to try to increase chip production by 10% per year for many years to come.

    Which is dynamic, Case A or Case B? Explain why you chose the one you

say is dynamic. Explain why the other one is not dynamic.

15. `[Problem ID #15] dynamics: classifying cases`
A farmer is raising tomatoes. It takes about 10 weeks to raise tomatoes. They must be harvested just before they are ripe, because they continue to get ripe while being shipped. So depending on the distance to the market, the farmer will harvest a little earlier or later. Is this a *dynamic* problem? Explain why or why not.

16. `[Problem ID #16] dynamics: classifying cases`
Student A finds that he cannot take Economic Dynamics because Macroeconomics, a course required for graduation, is taught at the same time. Student B finds that she cannot take Economics Dynamics because she has not completed Microeconomics, which is set as a prerequisite for Economic dynamics. Which student faces the more *dynamic* constraint? Explain your choice.

17. `[Problem ID #17] dynamics: irreversibility, example`
Give an example of an *irreversible* process in the economy, and explain the relationship of irreversibility to economic dynamics.

18. `[Problem ID #18] dynamics: modeling`
Consider a student's total number of credits earned as a *state variable*.

(a) Explain how this state variable helps you in dynamically planning your academic career (*e.g.*, choice of courses).

(b) Do you think a plan that says, "when you have X credits, take courses A, B, and C," is a sensible plan? Explain why or why not.

19. `[Problem ID #19] dynamics: modeling`
Consider a student's *grade point average* (GPA) as a *state variable*. GPA first converts the grade earned in each course to *grade points* (GP) using the scale A = 4, B = 3, C = 2, D = 1, F = 0. Then each grade is weighted by the number of credits earned for the course. Finally, the sum of weighted GP is divided by the total number of credits:

$$\text{GPA} = \frac{\sum_{i=1}^{n} \text{credits}_i GP(\text{grade}_i)}{\sum_{i=1}^{n} \text{credits}_i}.$$

(a) Explain how this state variable helps you in dynamically planning your academic career (*e.g.*, choice of courses).

(b) Do you think a plan that says, "when you have a GPA of X, take courses $SE100$, $ME200$, and $CRP300$," is a sensible plan? Explain why or why not.

20. `[Problem ID #20] dynamics: vs. microeconomics; scarcity definition`

What is a *scarce resource* in microeconomics? Be sure to define both "scarce" and "resource."

21. [Problem ID #21] dynamics: vs. microeconomics; allocation definition

    What is *allocation* in microeconomics? How is allocation related to *opportunity cost*?

22. [Problem ID #22] innovation: technology; R&D racing, overproduction

    What is the socially undesirable result of "R&D racing," for example to be the first to patent a new medical drug?

23. [Problem ID #23] growth: Solow; basic abstraction: no bond market

    Some macroeconomic models include the *bond market* as a source of finance for the government deficit. Why does Solow's model omit the bond market?
    マクロ経済学モデルに政府赤字を埋めるために国債を導入する。ソローモデルに国債を無視する理由を説明せよ。

24. [Problem ID #24] growth: Solow; basic abstractions

    The money supply is *not* considered in Solow's model.

    (a) What assumption about the economy is implied by this?

(b) Why do you think Solow left this variable out of his model?

25. `[Problem ID #25] growth: Solow; notation`
Consider *Solow's growth model*. Write an expression expressing the "rate of saving per worker."

26. `[Problem ID #26] growth: Solow; steady state definition`
Consider *Solow's growth model*. Write the equation *defining* the steady state.

27. `[Problem ID #27] growth: Solow; phase diagram solution`
The graph in Fig. 1 is used to analyze Solow's growth model.

(a) Label the axes, important intersection points, and curves with appropriate variables and functions.

(b) Find k^* and plot it on the graph.

(c) Sketch the curve representing $\dot{k}$ as a function of k. Overall, the curve can be quite approximate, but intersections defining steady states must be accurate.

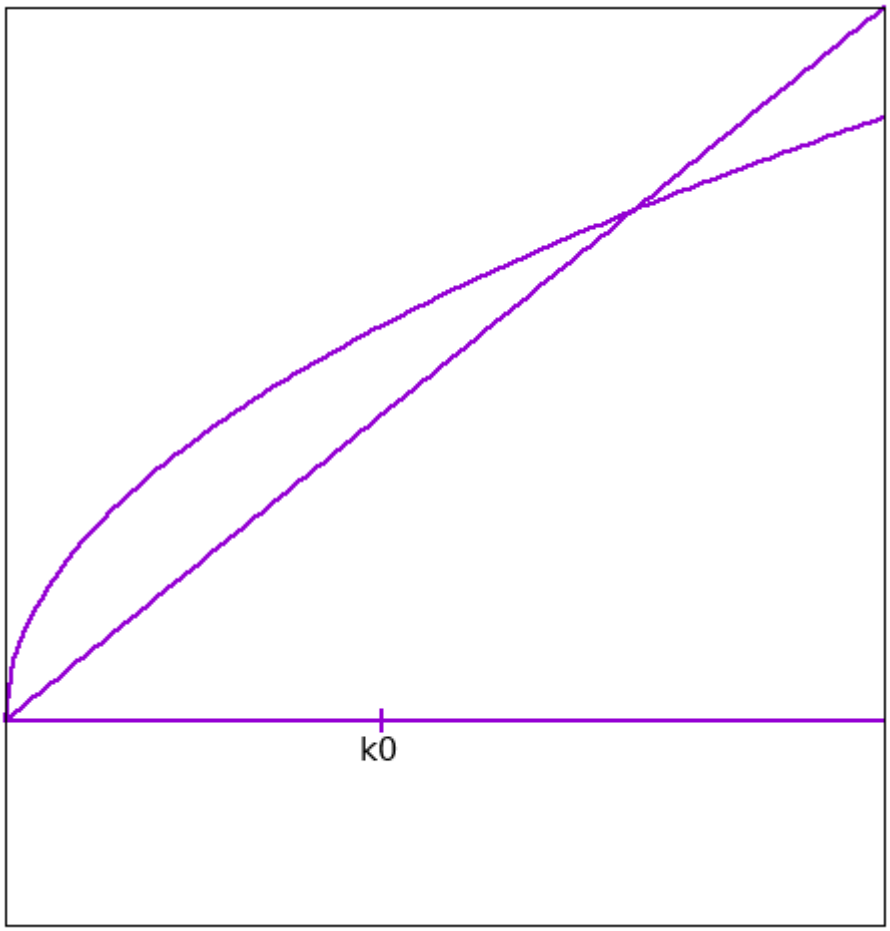
(d) If k starts at $\hat{k}$ now, what can you say about its future behavior?

28. `[Problem ID #28] growth: Solow; phase diagram solution identical to previous (up to graph)`
The following graph is used to analyze Solow's growth model.

(a) Label the axes, important intersection points, and curves with appropriate variables and functions.

(b) Find k^* and plot it on the graph.

Figure 1: As in exam



(c) Give the simplest possible expression describing $\dot{k}$ in *steady state balanced growth*.

29. `[Problem ID #29] growth: Solow; neutral progress`
Recall that in the Solow model with (Harrod-neutral) technological progress, the characteristic equation is $\dot{k} = sf(k) - (n + d + \lambda)k$. Name and briefly describe each of the following symbols from the characteristic equation:

（ハロッド中立）技術進歩を含むソーロモデルでは基本方程式は $\dot{k} = sf(k) - (n + d + \lambda)k$ だ。以下の記号の名前に短い説明を書け。

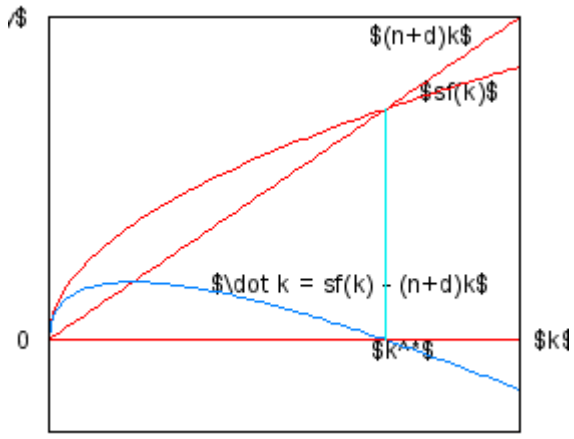


Figure 2: Solution

(a) k

(b) s

(c) n

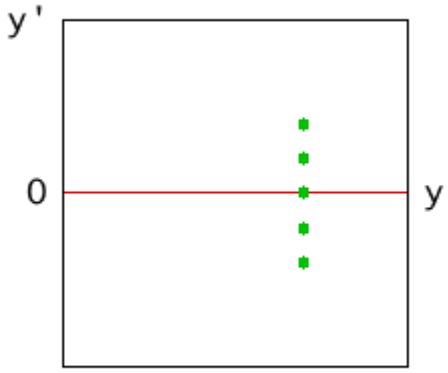
(d) λ

(e) What is the **steady state growth rate** of per capita consumption $c = \frac{C}{L}$?
一人当りの消費 $c = \frac{C}{L}$ の**定常状態での成長率**を書け。

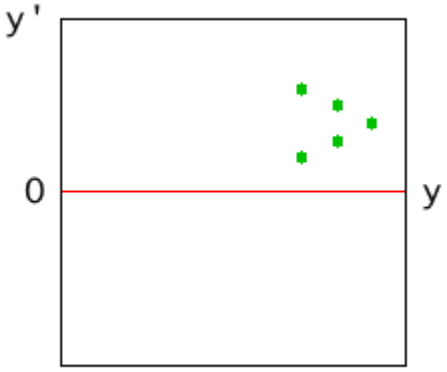
30. `[Problem ID #30] growth: Solow; convergence: graphical description`

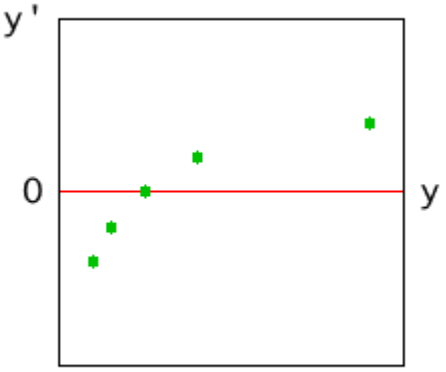
Suppose that a group of countries recently showed the pattern in the graph below for *income growth rate vs. level of income*.

(a) Which of the following graphs represents the most likely state of convergence (or failure to converge) 30 years hence?

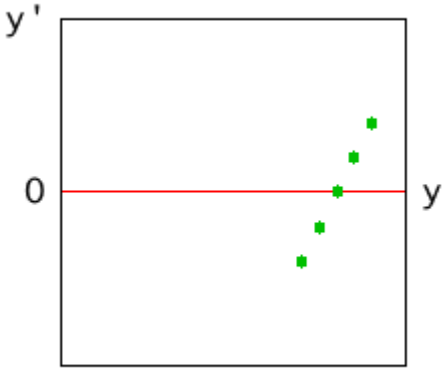


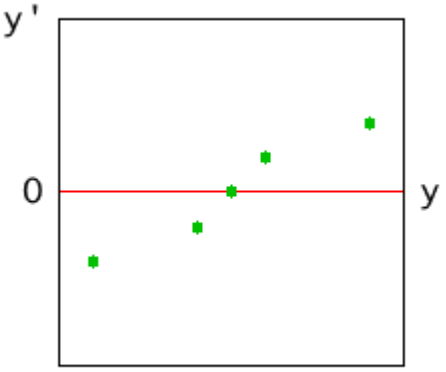
A B





C D





(b) Explain your answer.

31. `[Problem ID #31] growth: Solow; convergence: graphical description`

Each of the graphs in Fig. 3 shows a group of three countries, plotting their *per capita* income and their *per capita* income growth rate. One will display convergence of fast-growing low-income countries to the status of the high income country, the other will not.

(a) Label each graph as "Converging" or "Not Converging".

(b) On each graph, plot the likely positions of the countries after ten years (remember, the highest income country always has the value 1 because income is measured as a fraction of that country's income).

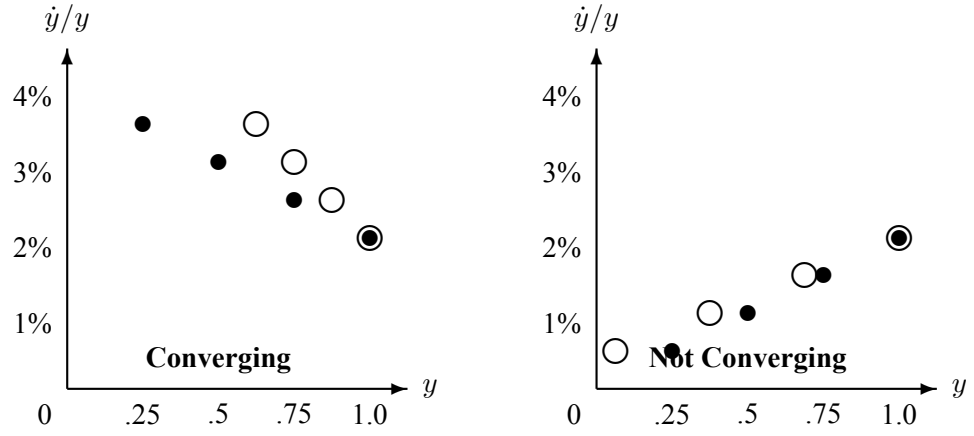


Figure 3: Convergence examples

Note: to get credit for the graph, you just need to put all the points to right of the original line of dots for the "Converging" case and to the left for the "Not Converging" case. For full credit, the highest income point should be the same before and after.

32. `[Problem ID #32] steady state: vs equilibrium`
What is the difference between *steady state* and an *equilibrium*? (Keep your answer as simple as possible.)

33. `[Problem ID #33] steady state: vs equilibrium`
Why does dynamic analyis focus on *steady state* instead of *equilibrium*?

34. `[Problem ID #34] growth: exponential computation`
Assume constant savings rate of s in the Solow growth model, and a TFP growth rate of 2%. After one generation (32 years) of balanced steady state growth, what is the per capita consumption?

35. `[Problem ID #35] dynamics: discounting computation`
Suppose the interest rate is $\delta = \frac{1}{3}$ (33.3%). Compute the *present discounted value* of 100,000 yen received in two years' time.

36. `[Problem ID #36] production function: CRTS: induction from examples`
Consider three different production functions F_a, F_b, and F_c. For each case i below, (1) answer whether the production function F_i appears to satisfy constant returns to scale (CRTS), and (2) justify your answer (*i.e.*, explain why or why not).

    (a) $F_a(12, 30) = 54$ and $F_a(4, 10) = 18$.

    (b) $F_b(19, 6) = 32$ and $F_b(38, 12) = 48$.

(c) $F_c(3,4) = 5$ and $F_c(12,8) = 15$.

37. `[Problem ID #37] growth: history of thought`
Match each school of economics with the factor of production that it emphasizes as the source of growth or wealth, and the explanation of why the factor was considered most important by that school.
Please fill in your answers below, in lines 1–5. Please do not write out the answers, but use the labels A–E.
Note that in the table of alternatives, each column is in alphabetical order, so there is no logical connection in the order here.

38. `[Problem ID #38] production function: CRTS: definition`
Give the definition of *constant returns to scale*. Show that the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$ satisfies constant returns to scale.

39. `[Problem ID #39] steady state: definition: Solow`
Give the definition of *steady state* in dynamics. What is the equation defining steady state in Solow's growth model?

40. `[Problem ID #40] growth: Solow; basic abstraction: constant n`
The labor force growth rate is *made constant* in Solow's growth model.
ソーロの経済成長モデルでは労働人口の成長率が定数である。

(a) What assumption about the economy is implied by this?
これを考えればはどういう経済に関する仮定に導かれる？

Label	School	Factor	Explanation
Alternatives			
A	Ancient (Greeks, Romans)	Capital	History has many examples of countries getting rich by going to war and taking this factor from other countries.
B	Classical (Smith, Marx)	Labor	Not really a factor of production, the school that concentrated on this idea believed that wealth came from God and was concerned with "fairness."
C	Neoclassical (Solow)	Land	This factor can be accumulated by allocating resources to it, and therefore is the natural focus of a dynamic theory of economic growth.
D	Physiocrat	Price	This factor is naturally productive alone, and there are limits to how much Man can improve its productivity.
E	Mediæval	Treasure	This human factor can be made more productive by specialization and trade, so its allocation is naturally the central issue of economics.
Answers			
1	A	E	A
2	B	B	E
3	C	A	C
4	D	C	D
5	E	D	B

(b) This assumption is *false* in the real economy. Explain why.
実際経済ではその仮定と違う。その理由を説明せよ。

(c) Why do you think Solow made this assumption anyway?
違うのにソーロがその仮定を立てた。その理由を説明せよ。

41. `[Problem ID #41] growth: Solow; steady state computation`
Consider a Solow model with the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$, a saving rate of $s = 0.2$, a depreciation rate of $d = 0.1$, and a labor force growth rate of $n = 0$.

(a) What is the growth rate of Y in steady state? Explain how you know without solving the model.

(b) Suppose the initial population is $L_0 = 1000$. Solve the model, giving expressions for all variables as both macro variables and micro (per worker) values. Don't forget consumption!

42. `[Problem ID #42] growth: Solow; convergence: draw OECD freehand`

Consider the *convergence hypothesis* for the OECD, which *does* show historical evidence for the convergence hypothesis.

(a) Give the convergence hypothesis.
収束仮説を書け。

(b) Explain why the convergence hypothesis seems likely to be true, referring to the parameters of the characteristic differential equation.
なぜ収束仮説があいろうかを説明せよ。成長の微分方程式のパラメーターに触れること。

(c) Draw "before" and "after" graphs of growth rate ($\dot{y}/y$) and income (y, where the unit is US real per-capita income in each period) for four countries whose situation is similar to the OECD.
OECDの状況を似ている４ヶ国の成長率 ($\dot{y}/y$) 対所得 (y、ただし単位が米一人当り実質所得) を書け。

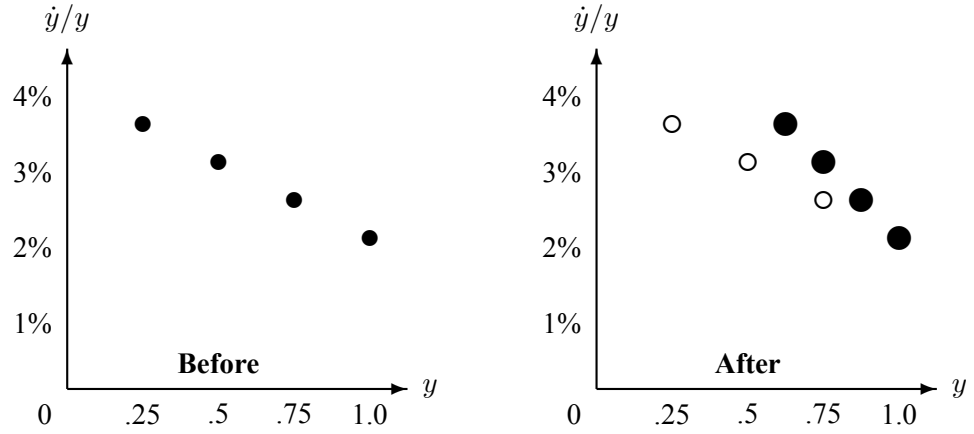


Figure 4: Convergence examples

43. [Problem ID #43] dynamics: classifying cases: GPA
Do the previous problem for a state variable defined as the grade point average (GPA). (GPA first converts the grade earned in each course to a number by the scale A = 4, B = 3, C = 2, D = 1, F = 0. Then each grade is weighted by the number of credits earned for the course. Finally, the sum of weighted GP is divided by the total number of credits:

$$\text{GPA} = \frac{\sum_{i=1}^{n} \text{credits}_i GP(\text{grade}_i)}{\sum_{i=1}^{n} \text{credits}_i}.$$

44. [Problem ID #44] growth: Solow; basic abstraction: constant

`s`
The saving rate as a fraction of income is *made constant* in Solow's growth model.

(a) What assumption about the economy is implied by this?

(b) This assumption is *false* in the real economy. Explain why.

(c) Why do you think Solow made this assumption anyway?

45. `[Problem ID #45] growth: Solow; role of technology`
Understanding the role of technological progress in economic growth is very important.

(a) In economics, technological progress is represented by an increase in output from constant factor inputs. There are three special representations of technological progress: *Harrod neutral technological progress*, which increases the productivity of labor, *Solow neutral technological progress*, which increases the productivity of capital, and *Hicks neutral technological progress*, which increases the productivity of both factors by the same amount. Which is most useful in adapting Solow's model to include technological progress? Explain briefly.

(b) What is the *characteristic differential equation* for the Solow model with technological progress?

$$\dot{\tilde{k}} = sf(\tilde{k}) - (n + d + \lambda)\tilde{k}.$$

(c) In this model, although the capital to effective labor ratio is constant in steady state, and so output per effective labor is constant, the consump-

tion per worker increases steadily. Explain.

46. `[Problem ID #46] growth: Solow; convergence: false for whole world`
Consider the *convergence hypothesis* for the whole world, which *does not* show historical evidence for the convergence hypothesis.
全世界における収束仮説を考え、特に歴史的な根拠がないことを。

(a) Give the convergence hypothesis.
収束仮説を説明せよ。

(b) What is the evidence against the convergence hypothesis for the world? (It may be helpful, but is not necessary, to draw graphs to support your answer.)
全世界における収束仮説を否定する証拠を述べよ。（グラフが必要ではないが、使っても構わない。

(c) There are several assumptions about equality across countries of parameters of the Solow model that all must be true to support the convergence hypothesis. Pick one parameter of the model and explain why it seems not to be equal across countries, and might remain unequal.
収束仮説が成り立つためにモデルにあるいくつかのパラメーターが各国で同じ値にならなければならないのだ。その中のひとつを選んでそれぞれの国に違う値があると説明してその違いが永

久的に残る可能性への根拠を述べろ。

47. [Problem ID #47] growth: Solow; convergence: definition, example

Consider the idea of *convergence* in growth theory.
経済成長論における収束という概念を考えよう。

(a) Define *convergence*.
収束の定義を書け。

(b) Compare recent economic performance of Japan, South Korea, and North Korea as an examples of convergence or failure to converge.
日本、韓国、北朝鮮の近年の経済成長を取り上げて収束と非収束の例として述べろ。

48. [Problem ID #48] steady state: definition: fishery
Give the definition of *steady state* in dynamics. What is the equation defining steady state in the fishery model?

49. [Problem ID #49] production function; neoclassical conditions

There are three conditions that must be satisfied by the per-capita production function $f(k)$. For each condition:

- Give its name or explain its meaning in words.
- Give an equation or inequality defining the condition precisely.
- Show that the function $f(k) = k^{\frac{1}{2}}$ satisfies the condition.

50. `[Problem ID #50] exhaustible: renewable; natural rate of increase`

The natural rate of increase of a population $\dot{Z}$ which is not being harvested (*i.e.*, $y = 0$) is described by a function $H(Z)$, leading to the differential equation $\dot{Z} = H(Z)$.

(a) The logistic form $H(Z) = \beta Z(1 - \delta Z)$ is often used to describe a *renewable resource* (population) whose growth is constrained by some scarce resource. Explain how this mathematical form expresses that constraint.

(b) Give a formula for the H function that describes a *pure exhaustible resource* like oil.

(c) The formula for the H function for a "renewable resource" like solar power would be the same as for oil. This seems to be a paradox. Explain briefly how this can be. (Hint: how does consumption y affect the

”stock” of solar power?)

51. `[Problem ID #51] steady state: definition: fishery`
Consider the idea of *steady state*.
定常状態を考えよう。

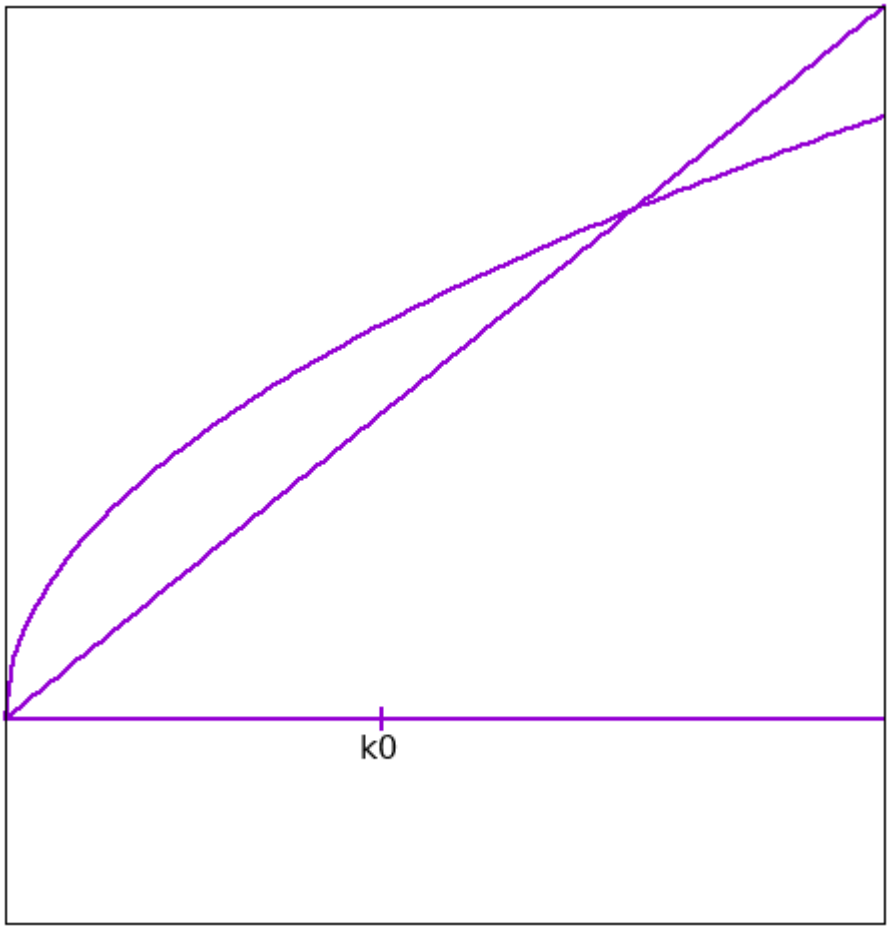
(a) Define *state variable* (in general).
状態変数の定義を書け。

(b) Define *steady state* (in general).
定常状態の定義を書け。

(c) What is the equation defining steady state in the fishery model? (Recall that the notation for the size of population is Z, for the law of natural increase is $H(Z)$, and for the harvest rate is y.)
漁業モデルでの定常状態条件 (式) を書け。

52. `[Problem ID #52] growth: Solow; phase diagram solution`
(20 points) Fig. 5 is the phase diagram of Solow's growth model.

Figure 5:



(a) Label the axes, important intersection points, and curves with appropriate variables and functions.

(b) Find k^* and plot it on the graph.

(c) On the graph in Fig. 6, show the effect of an *increase* in s on the stable steady state, sketching any additional curves. You do not need to reproduce all the labels from the diagram above, but you must use enough labels to make the effect clear.

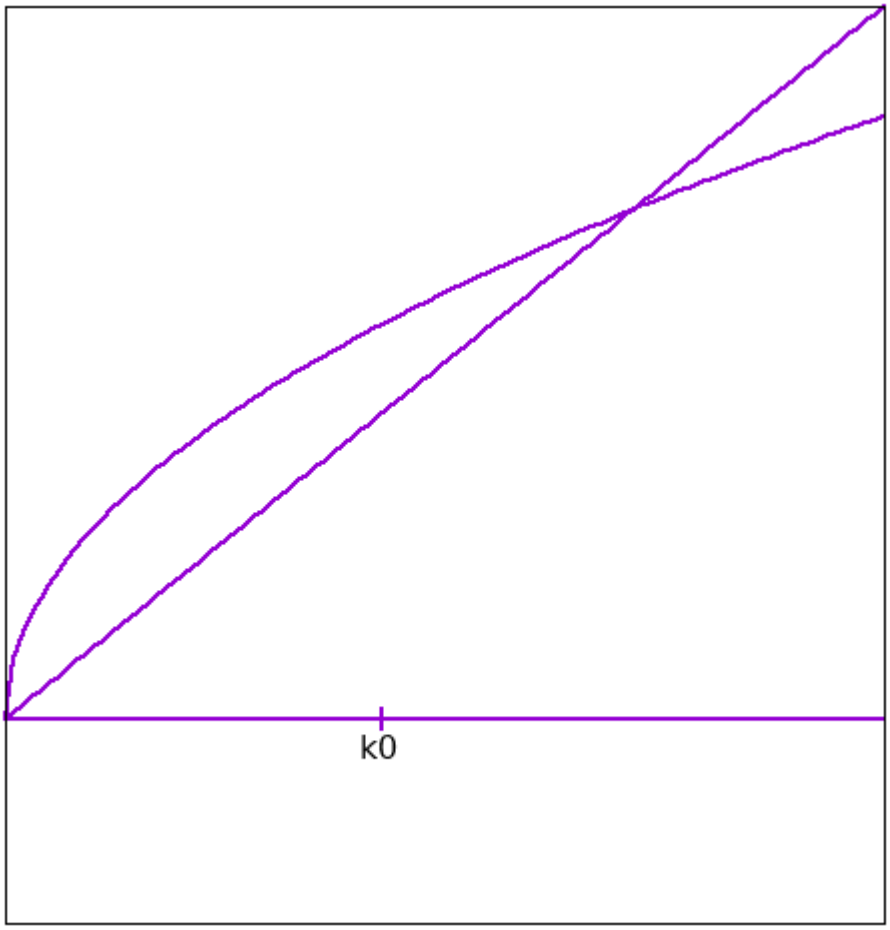


Figure 6: Comparative statics of an increase in s

53. `[Problem ID #53] optimization: dynamic programming; modeling`

(20 points) Many dynamic problems can be expressed as choosing a sequence of states $\{X_0, X_1, X_2, \ldots\}$ subject to a constraint $X_{t+1} \in \Gamma(X_t)$, giving current utility (or profit) $U(X_t, X_{t+1})$ in period t.

(a) The *Bellman equation*

$$\forall t : v_t(X_t) = \max_{X_{t+1}} U(X_t, X_{t+1}) + \rho v_{t+1}(X_{t+1})$$

is a condition that characterizes the optimal sequence of Xs. Explain

the meaning of v_{t+1}.

(b) In the Solow model, current utility can be represented simply by $c_t = C_t/L_t$, the per-capita consumption at time t. The law of motion in discrete time is $K_{t+1} - K_t = I_t - D_t$, where the variables are capital (K), gross investment (I), and depreciation (D), respectively. Give a formula for $U(X_t, X_{t+1})$ in this case.

(c) Can you write the formula for the previous question in terms of $k_t = K_t/L_t$ and k_{t+1} only, without using K_t and L_t separately? Explain why it is important to do so if possible.

54. `[Problem ID #54] diffeq: solve first order`
Verify that the function $P(t) = Ae^t$ is a solution to the differential equation $\frac{dP}{dt} = P$.

55. `[Problem ID #55] dynamics: classifying cases`
Consider the following two situations involving time. One is dynamic and the other is not.

- **Case A:** A copper-exporting country decides to try to increase copper production by 10% per year for many years to come.
- **Case B:** A country that provides anonymous numbered bank accounts decides to try to increase fee revenues by 10% per year for many years to come.

Which is dynamic, Case A or Case B? Explain why you chose the one you

say is dynamic. Explain why the other one is not dynamic.

56. [Problem ID #56] dynamics: classifying cases: finance
Explain why most models in the subject of finance may be considered dynamic.

57. [Problem ID #57] dynamics: classifying cases: experimental economics
In both real markets and experimental markets, "bubbles" in asset markets where market price rises above the fundamental value of the asset are observed. (The fundamental value is the sum of the dividends expected to be received from the asset.) Explain why this must be considered a dynamic phenomenon.

58. [Problem ID #58] exhaustible: renewable; empirical
In Spence's whaling model, for five years up to 1960 the number of whales harvested was nearly the same as, or greater than, the model's estimate of the whale population. What does this indicate about the accuracy of such models in extreme conditions? Assume that whale pregnancy is similar to humans in the length of time (typically it is greater) and number of calves per birth.
スペンスの捕鯨モデルで1960年までの5年間の間に取ったクジラの数と総頭 数がほぼ同じだった。こういうモデルの正確さについて何を示しているかに ついて述べろ。（クジラの妊娠過程とは人間のとよく似ている。）

59. `[Problem ID #59] diffeq: solve first order`
Verify that the function $P(t) = A + \frac{1}{2}Bt^2$ is a solution to the differential equation $\frac{dP}{dt} = Bt$.

60. `[Problem ID #60] exhaustible: Hotelling rule`
What is the path followed by the price of a *pure exhaustible resource*? Explain the roles of financial markets and resource market demand in determining this price.
純粋尽くせる資源の価格パスの特徴を説明せよ。金融市場と資源市場の需要の役割を含むこと。

61. `[Problem ID #61] exhaustible: renewable; natural rate of increase`

(20 points) Suppose the natural rate of increase of a population $\dot{Z}$ is described by the expression $H(Z) = -0.05(Z+1)(Z-19)$, leading to the differential equation $\dot{Z} = -0.05(Z+1)(Z-19)$.

(a) What are the three basic conditions for a curve to be called "bell-shaped"? Does $H(Z)$ satisfy them? Explain how $H(Z)$ satisfies or fails to satisfy each condition.

(b) What do the notations $\underline{Z}$, $\bar{Z}$, and $\hat{Z}$ refer to?

$\underline{Z}$ is

$\bar{Z}$ is

$\hat{Z}$ is

(c) Compute these three important values of population $\underline{Z}$, $\bar{Z}$, and $\hat{Z}$ for the H given above.

(d) Sketch the graph of $\dot{Z}$, clearly labelling the main parts of it.

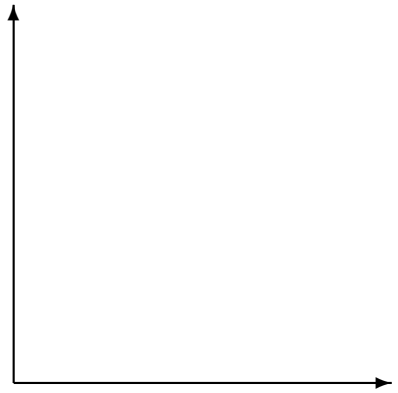


Figure 7: A natural resource (biological population)

(e) What is the *maximum* rate of increase $\dot{Z}_{max}$?

(f) This function *fails* to satisfy at least one of the conditions we gave for a model of a renewable biological resource. What is it? Explain briefly why this condition should be satisfied.

(g) In growth theory, we often use "Inada conditions." What is the purpose of the Inada conditions in growth theory? Why don't we need similar conditions when analyzing a renewable resource?

62. `[Problem ID #62] diffeq: define`
Define *differential equation*.

63. `[Problem ID #63] diffeq: reduce order`
Convert the *second-order* differential equation $\frac{d^2h}{dt^2} = -g$ to a system of two *first-order* differential equations. (When g = 9.8 m/s^2, this is the equation for free fall under the influence of Earth's gravity.)

64. `[Problem ID #64] diffeq: solve (2nd order)`
Consider the second-order differential equation $x'' = -g$. (When g = 9.8 m/s^2, this is the equation for an object in free fall under the influence of the Earth's gravity.)

(a) Show the derivation of the general solution to this equation.

(b) Explain how we might determine a particular value for any constants of integration you needed to introduce.

65. `[Problem ID #65] diffeq: solve (1st order)`
Consider the first-order differential equation $x' = ax$. (This is the standard model of *unconstrained population growth*.

(a) Show the derivation of the general solution to this equation.

(b) Explain how we might determine a particular value for any constants of integration you needed to introduce.

66. `[Problem ID #66] dynamics: stock vs. flow`
For definitions, use economic terms, not mathematics.

(a) Define *stock* as used in economic dynamics.

(b) Define *flow* as used in economic dynamics.

(c) Explain the mathematical relationship between stock and flow.

67. `[Problem ID #67] diffeq: solve numerical`
Use the Euler approximation to solve the differential equation $y' = -2x + 4$.

(a) Use the initial condition $y(0) = 0'$, over the range -2 to 4, with step size 1.

(b) Solve the equation by symbolic integration. How inaccurate is the numerical approximation at $x = 4$?

68. `[Problem ID #68] diffeq: isocline`
Compute the isocline $y' = 2$ for the differential equation $y' = 4 - 2x$.

69. `[Problem ID #69] diffeq: IFS; recursive`
Give the general recursive model of dynamic system with state variable x.

70. `[Problem ID #70] diffeq: IFS; recursive-to-difference`
Convert the recursive dynamic model $x_{t+1} = f(x_t)$ to difference equation form.

71. `[Problem ID #71] diffeq: IFS; difference-to-recursive`
Convert the dynamic model in difference equation form $\Delta x_t = g(x_t)$ to recursive equation form.

72. `[Problem ID #72] diffeq: IFS; recursive paths`
Consider a recursive equation model $x_{t+1} = f(x_t)$, where x_0 can take the two values $\{0, 1\}$. How many different sequences can arise in this model (for fixed f)?

73. `[Problem ID #73] innovation: IP merits demerits`
Explain the economic advantages and disadvantages of granting *intellectual property*.

74. `[Problem ID #74] exhaustible: renewable; natural rate of increase`

(20 points) Suppose the natural rate of increase of a population is described by the expression $H(Z) = -0.1(x - 18)(x + 2)$.

(a) What are the three basic conditions for a curve to be called "bell-shaped"? Does $H(Z)$ satisfy them? Explain how $H(Z)$ satisfies or fails to satisfy each condition.

(b) What do the notations $\underline{Z}$, $\bar{Z}$, and $\hat{Z}$ refer to?

$\underline{Z}$ is

$\bar{Z}$ is

$\hat{Z}$ is

(c) Compute these three important values of population $\underline{Z}$, $\bar{Z}$, and $\hat{Z}$ for

the H given above.

(d) Sketch the graph of $\dot{Z}$. Don't forget to label axes, curves and important intersections.

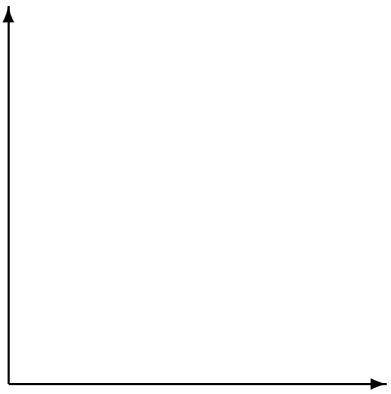


Figure 8: A natural resource (biological population)

(e) What is the *maximum* rate of increase $\dot{Z}_{max}$?

(f) This function *fails* to satisfy at least one of the conditions we gave for a model of a renewable biological resource. What is it? Explain briefly why this condition should be satisfied.

(g) Write the differential equation describing the growth of the population Z.

(h) In growth theory, we often use "Inada conditions." What is the purpose of the Inada conditions in growth theory? We don't we need similar conditions when analyzing a renewable resource. Explain why not.

75. `[Problem ID #75] steady state: definition: fishery`
Give the definition of *state variable* and *steady state* in dynamics. What is the equation defining steady state in the fishery model with harvesting? (Recall that the notation for the size of population is Z, for the law of natural increase is $H(Z)$, and for the harvest rate is y.)

76. `[Problem ID #76] diffeq: solve first order`
Verify that the function $P(t) = At$ is a solution to the differential equation $\frac{dP}{dt} = A$.

77. `[Problem ID #77] dynamics: vs. microeconomics; equilibrium, steady state`

(a) Define *equilibrium* as in microeconomics.

(b) Define *steady state* as used in dynamics.

(c) How are these two concepts different?

78. `[Problem ID #78] dynamics: vs. static`
The Japanese sales tax will be raised from 5% to 8% next April.

(a) What *static effect* do you expect on consumption from this change?

(b) What *dynamic effect* do you expect from this change?

79. `[Problem ID #79] dynamics: and economics; growth vs. fishery`

The Solow model of economic growth may be said to be a *pure dynamic model*, while the simple model of the fishery with harvesting does contain some *microeconomic content*. What is the important difference between these two models?
ソーロモデルは純粋動学的モデルと言われているが漁業のモデルはミクロ経済学的内容を含むと言える。重要な違いはどこにあるか。

80. [Problem ID #80] optimization: backward induction: computation

Fill in the following table for backward induction. The starting balance is 0, the target balance at the end of three years is 30000. Utility is logarithmic ($u(C) = \ln C$). The discount factor is $\delta = 1$ in all periods.
(You do not need to evaluate the utility function; leave it in the form "$\ln 10000$" when consumption is 10000, for example.)

Year (t)	**1**	**2**	**3**
Income (Y)	10000	20000	30000
Saving ($S = s_t^*(X_{t-1})$)			
Consumption (C)			
Current utility ($u(C)$)			
Ending balance (X)			
Future value ($V_t = V_t^*(X_{t-1})$)			
Optimal saving ($s_t^*(X_{t-1})$)			
Value function ($V_t^*(X_{t-1})$)			

81. [Problem ID #81] growth: Solow; steady state computation
Consider a Solow model with the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$, a saving rate of $s = 0.2$, a depreciation rate of $d = 0.1$, and a labor force growth rate of $n = 0.03$. What is the growth rate of Y in steady state? Explain how you know without solving the model.

82. [Problem ID #82] exhaustible: asset arbitrage, Hotelling Rule

Following steps (a)–(c), derive the *Hotelling Rule* that $\dot{P} = rP$ from the arbitrage condition that the markets for both bonds and the stock of an exhaustible resource be in equilibrium. Then answer (d).

下記の (a)–(c) に従い、さやとり条件で $\dot{P} = rP$ のいわゆるホーテリングルールを導き出す。

(a) Formulate the *arbitrage equation* that defines asset market equilibrium in terms of the interest rate, current price of the exhaustible resource, and future price of the exhaustible resource, for a short period of time.
資産市場均衡を表すさやとり条件を利子率と、尽くせる資源の現価格と将来価格で組む。

(b) Why are markets that don't satisfy the condition not in equilibrium?
この条件を満たせなければ均衡にならないことを説明せよ。

(c) Show that by manipulating the equation and taking the limit as the period that the assets are held goes to zero, the Hotelling Rule for the price of the exhaustible resource can be expressed as an equation involving the growth rate of price and the interest rate.
式を変換し、資産を持つ期間をゼロに収束するリミットをとり、ホーテリングルールを導き出す。

(d) Explain why the Hotelling Rule implies no steady state for the exhaustible resource.
定常状態が存在しないことをホーテリングルールで説明せよ。

83. `[Problem ID #83] optimization: value function; innovation`
In their innovation model, Boldrin and Levine show the existence of a *value function* $v(k)$ where k is the stock of items implementing the innovation.
ボルドリンとレビーンの技術革新モデルではある値関数 $v(k)$ が存在する。

(a) How can you compute the competitive asset price for an item? (That is, suppose the item is to be used only for copying.)
市場での財産価格の計算を示すこと。

(b) Consider an innovation where the uses of consuming and copying are *rival* (if you consume, you can't copy, and if you copy, you can't consume). Suppose you have a stock of k items. Assume it is optimal for you to keep one item for consumption, which gives you a consumption value of u. You use the rest for copying where you have βx copies next period if you use x items for copying this period. Assume copying costs nothing. Write the equation which determines a stationary value function.

84. `[Problem ID #84] innovation: nonexistence of spillovers`
Boldrin and Levine claim that there are no spillovers of intellectual assets, even though they are non-rival.

(a) What is a *spillover* from an idea?

(b) Define the concept of *non-rival* good, and explain why ideas might be non-rival.

(c) Explain why a non-rival good might not provide any spillovers.

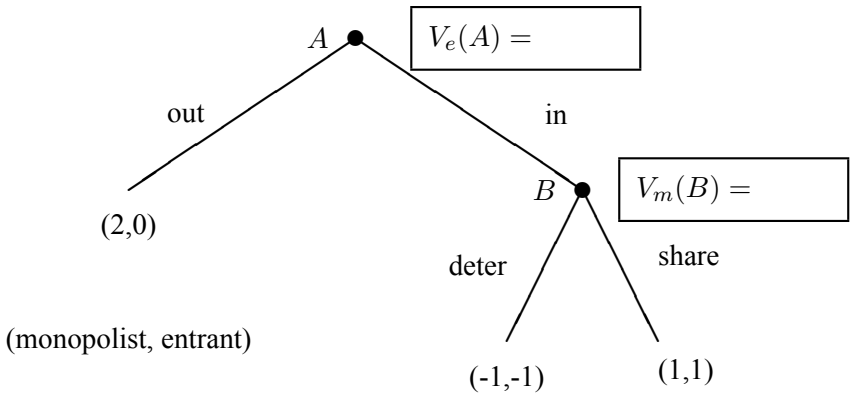
85. `[Problem ID #85] optimization: value function; extensive game`

Consider the extensive game in the diagram below.

(a) On the diagram enter the value of arriving at point A to the entrant, and point B to the monopolist based on the backward induction solution to the game.

(b) What are the equilibrium actions of the monopolist and the entrant?

(c) What are the equilibrium payoffs to the monopolist and the entrant?



86. `[Problem ID #86] dynamics: vs. static`
Prime Minister Abe announced a few weeks ago that the Japanese sales tax increase (from 8% to 10%, scheduled for April 2015, is postponed at least one year.

(a) What *static effect* do you expect on consumption in **April 2015** from the tax increase?

(b) What *dynamic effect* do you expect **in the housing market** from this change? Remember that it often takes about 3–5 months to actually complete an intended purchase of a home, and up to a year if a home is being newly built.

87. `[Problem ID #87] diffeq: solve first order`
Verify that the function $P(t) = A$ is a solution to the differential equation $\frac{dP}{dt} = 0$.

88. `[Problem ID #88] chaos: vs. randomness`
Consider *chaos* and *randomness*.

(a) How are these two concepts *similar*?

(b) How are these two concepts *different*?

89. `[Problem ID #89] optimization: value function; extensive game`

Consider the extensive game in Fig. 9, which describes a professor advising a master student writing his thesis. She wants the student to do publishable research and go on to get a Ph.D., but he just wants to graduate with minimum effort and get a job as research staff in a big company.

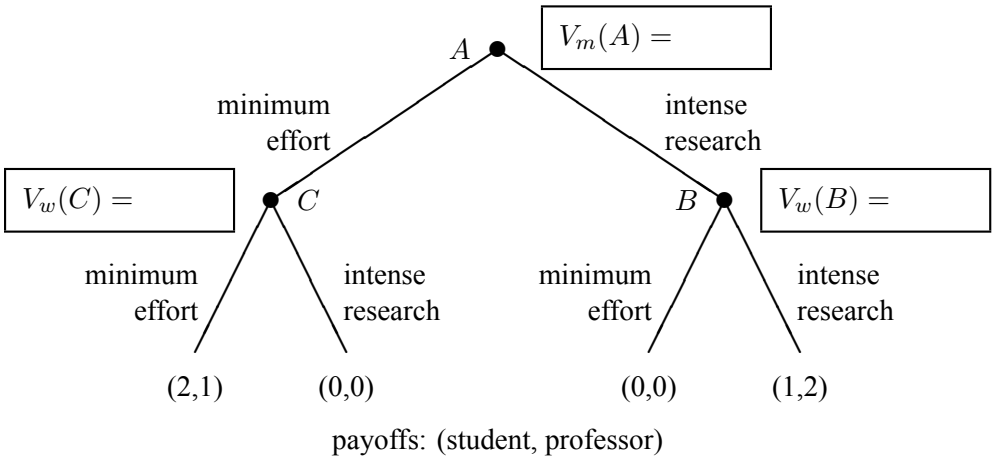


Figure 9: Battle of the Sexes

(a) Explain how the payoffs reflect the preferences of the professor and her student. In particular, what do the $(0, 0)$ entries mean?

(b) On the diagram enter the value of arriving at point A to the student, and points B and C to the professor based on the backward induction solution to the game.

(c) What are the equilibrium actions of the professor and the student?

(d) What are the equilibrium payoffs to the professor and the student?

(e) In this question, the student "acts first." Do you think this is more believable or less believable than having the professor act first? Explain why you think so.

90. `[Problem ID #90] dynamics: vs. static`
The Japanese sales tax increase (from 8% to 10%, originally scheduled for April 2015, will be imposed in April 2016.

(a) What *static effect* do you expect on consumption in **April 2016** from postponing the tax increase?

(b) What *dynamic effect* do you expect **in the housing market** from this change? Remember that it often takes about 3–5 months to actually complete an intended purchase of a home, and up to a year if a home is

being newly built.

91. `[Problem ID #91] diffeq: solve first order`
Verify that $P(t) = Ae^{\lambda t}$ is a solution to the differential equation $\frac{dP}{dt} = \lambda P$.
$P(t) = Ae^{\lambda t}$ が微分方程式 $\frac{dP}{dt} = \lambda P$ の解であることを確認せよ。

92. `[Problem ID #92] production function: CRTS in Solow model`
Constant returns to scale (CRTS) in the production function is an important assumption of the Solow model of economic growth.
生産関数がCRTSであることはソーロ経済成長モデルの重要な前提だ。

(a) Using the assumption of CRTS allows Solow to transform a standard macro model into simple differential equation. Explain what transformation Solow used that depends on the assumption of CRTS.
ソーロはCRTSを用いて標準的マクロモデルを簡単な微分方程式に変換できるようにした。その変換法を説明せよ。

(b) Explain why this transformation is a good fit to the way that modern economists think about analyzing social problems.
この変換法が現代経済学者の社会問題の分析に関する考え方はよく合っていると言える。どう合っているか説明せよ。

93. `[Problem ID #93] dynamics: vs. microeconomics; equilibrium, steady state`
In intermediate economics, *equilibrium* is defined as a state in which all variables take on values so that no agent has an incentive to change behavior. Economic dynamics takes a different approach.

入門経済学では「均衡」が「全ての変数の値のもとでは行動を変更したい主体がいない」と定義されている。経済動学の観点が違う。

(a) Define *steady state* as used in dynamics.
動学で「定常状態」の定義を書け。

(b) Explain why dynamics focuses on steady state rather than equilibrium. (Hint: the Solow model of economic growth may be a good example.)
動学が均衡ではなくて定常状態を焦点にする理由を説明せよ。

94. `[Problem ID #94] exhaustible: simple fishery; phase diagram analysis; logistic`
(30 points) We discussed the *fishery* model of a single population as renewable resource with constraints on growth. Consider the *logistic model* of population growth:

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z).$$

漁業での魚母団の成長制限された再生可能資源としたモデルを講義で触 れた。母団成長のロジスティックモデルを検討しよう：

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z)$$

(a) What do the expressions a and bZ represent? (Hint: it may be useful to expand the expression given for $H(Z)$.)
a と bZ という式は何を表現するか。（ヒント： $H(Z)$ の別の式が役立つかも。）

(b) What can you say about the case $b = 0$?
$b = 0$ の場合には何が言えるか。

(c) Show that H satisfies the four conditions for a *bell curve*.
H がベル曲線の 4 条件を満たすことを証明せよ。

(d) Solve for the populations $\underline{Z}$ and $\bar{Z}$ such that $H(\underline{Z}) = H(\bar{Z}) = 0$.
母団数 $\underline{Z}$ と $\bar{Z}$ は $H(\underline{Z}) = H(\bar{Z}) = 0$ を満たす。 $\underline{Z}$ と $\bar{Z}$ の解を求めよ。

(e) What is special about the populations $\underline{Z}$ and $\bar{Z}$?
母団数 $\underline{Z}$ と $\bar{Z}$ の特徴を説明せよ。

(f) Solve for the population Z^* such that $H'(Z^*) = 0$.
$H'(Z^*) = 0$ を満たす母団数 Z^* の解を求めよ。

Now consider the basic model of a constant level of extraction $y \geq 0$:

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z) - y.$$

(This is also called an "open loop" dynamic system.)
収穫を定数 $y \geq 0$ で常に取る基本的モデルを考えよう：

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z) - y$$

（オープンループシステムとも言われる。）

(g) Consider the set of *steady states* of this system. The populations in this set change gradually as y increases from 0, and at certain "sufficiently high" levels the set itself changes dramatically. At what level or levels do the "dramatic" changes in the set occur?

定常状態の集合を考えよう。集合の母団数がyのゼロからの増加によって連続的に変動するが、特定な「高い」水準が集合の形が急に変わる。その水準を求めよ。

(h) Solve for the set of steady states, taking care to account for the "dramatic" changes as y increases.
yの増加による集合の形の変化を配慮しながら定常状態集合を求めよ。

(i) Which steady states are stable? Which are unstable?
安定定常状態と不安定定常状態を書け。

(j) On the graph below, sketch H and its relation to y. Add labels to indicate the various important parts of the graph. (Don't forget the axes!)
以下のグラフに H と、 y と H との関係をおおよそに描け。注目すべく部分にラベルを振ること。（軸も忘れずに！）

95. `[Problem ID #95] innovation: free consumption case`
In the Boldrin-Levine model with $\zeta = \beta$, explain in words why $c_t = k_t$.

96. `[Problem ID #96] innovation: Boldrin-Levine law of motion`
Boldrin and Levine model the spread of inventions by assuming that it takes time to buy a copy of a new invention and then make and sell further copies. They derive a law of motion, the iterated function equation

$$k_{t+1} = \beta(k_t - c_t) + \zeta c_t.$$

Explain each of the following symbols in the equation:

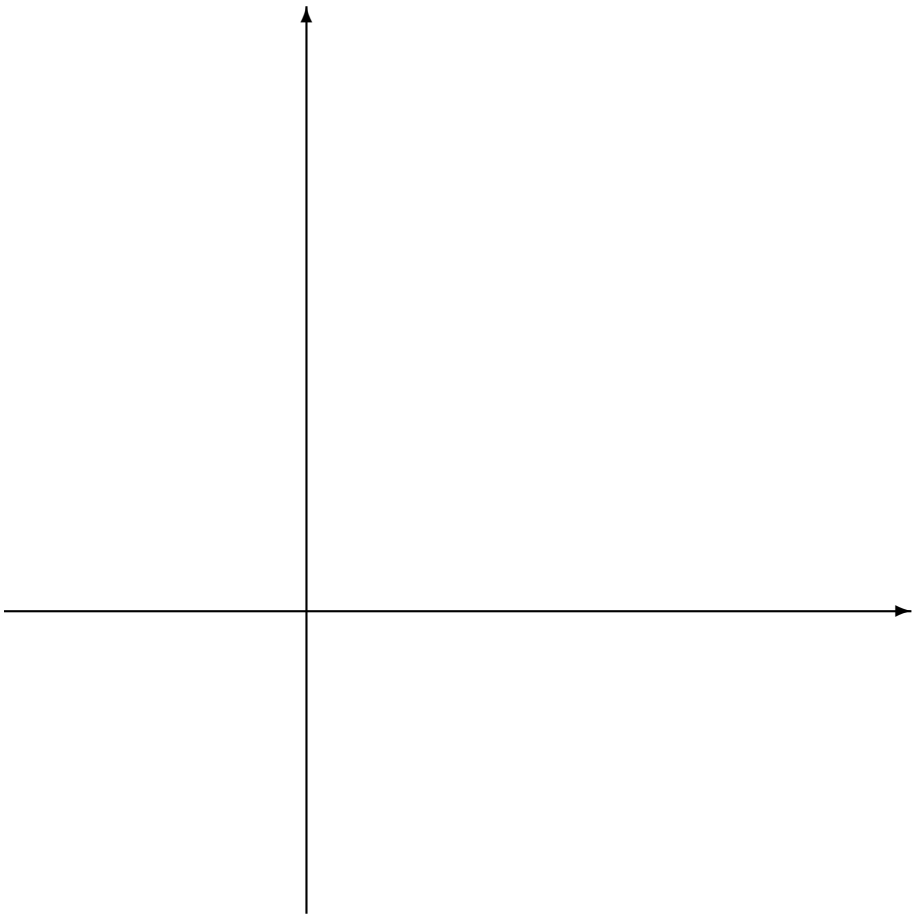


Figure 10: Phase diagram of the fishery with harvesting

(a) t

(b) k

(c) c

(d) β

(e) ζ

97. `[Problem ID #97] innovation: value function`
Boldrin and Levine model the spread of inventions by assuming that it takes time to buy a copy of a new invention and then make and sell further copies. They solve their model by defining a series of functions

$$v_t(k) = \max_c u(c) + \delta v_{t+1}(\beta(k-c) + \zeta c).$$

(a) What do we call a function like $v_t(k)$?

(b) Give the equation that defines a *stationary solution* to the problem.

(c) Explain the meaning of the equation $p_t = u'(c_t)$ in economic terms.

(d) Explain the meaning of the equation $q_t = v_t'(k_t)$ in economic terms.

(e) What equation characterizes market equilibrium in period t?

98. `[Problem ID #98] chaos: characterization`
Characterize each of the following properties that a chaotic dynamic process might have as *true* or *not true*. For those that are *not* true, explain why not. (The majority of credit depends on your explanation of *why not*. A correct explanation can be a short as one brief sentence for each answer, but you may use as many words as you need.)
カオスプロセスの特徴として考えられる、以下のことについて「ある」か「ない」と答えた上、ないの場合には説明せよ。

(a) cannot be predicted accurately in the (sufficiently) far future.
長い将来なら正確に予測できない。

(b) has no fixed points or cycles.
サイクルがない。

(c) is random.
ランダムである。

(d) will fill the entire space between its maximum and minimum limits, if you try enough different starting points.
初期の値を変動させれば最低値と最高値の間のすべてのポイントに当 たる。

99. `[Problem ID #99] chaos: vs. randomness`
Explain how statistics can distinguish between a *random* process and a *chaotic* process.

100. `[Problem ID #100] innovation: no spillovers`
Boldrin and Levine claim that there are no *spillovers* in innovation.

(a) What is a spillover?

(b) Why do Boldrin and Levine say there are no spillovers?

(c) Solow's model with labor-enhancing technological progress as usually presented does have spillovers. How do you know that technological progress in Solow's model have spillovers?

(d) The characteristic equation of Solow's model with labor-enhancing tech-

nological progress is

$$\dot{k} = sf(k) - (n + d + \lambda)k.$$

Which parameter (symbol) in this equation represents the spillovers of technical progress?

(e) (5 points extra credit) In fact, Solow's basic model (without techological progress) already has a spillover of a different kind in it. What is this spillover?

101. `[Problem ID #101] innovation: creative destruction`
What is *creative destruction* in economic growth?

102. `[Problem ID #102] innovation: spillover and market failure`

"Spillovers" of technological progress (innovation) from innovators to other actors in society are believed to create a *market failure*. Describe this market failure.

103. `[Problem ID #103] A`
ccording to theinnovation: second best "theory of the second best," combatting the market failure associated with technological progress (innovation) may require creating an additional market failure. Name the policies most countries use to repair the first market failure, and explain how they result in the second market failure.

セコンドベスト理論により、技術進歩 による市場の失敗を直すには他の市場の失敗を認めなければならない場合がある。前者の失敗に対応するためのよく使われた対策とその後者への効果を説明せよ。

104. `[Problem ID #104] innovation: as dynamics`
Explain why *innovation* is by definition a dynamic process.

105. `[Problem ID #105] innovation: vs. invention`
Economists typically differentiate between *invention* and *innovation*. What is the difference between these two social processes, and why is it important in economics?

106. `[Problem ID #106] innovation: vs. invention`
Economists typically differentiate between *invention* and *innovation*.
経済学では「発明」と「技術革新」を区別する。

(a) Explain the difference between these two social processes.
その違いを説明せよ。

(b) Explain why this difference is important in economics.
経済学ではその違いの重要な意義を説明せよ。

(c) Explain why *innovation* is by definition a dynamic process.
技術革新があくまで動学的であることの理由を説明せよ。

107. `[Problem ID #107] production function: CRTS computation`
A neoclassical, constant returns to scale (CRTS) production function is a crucial assumption in growth theory.
ネクラシカルCRTS生産関数は経済成長論に不可欠な前提です。

(a) For a generic production function $F(K, L)$, write the condition it must satisfy to be considered CRTS.
一般的生産関数$F(K, L)$がCRTSになる条件を書け。

(b) For the Cobb-Douglas production function, $F(K, L) = AK^{\alpha}L^{1-\alpha}$, derive the per-capita production function $f(k)$.
カブダグラス生産関数$F(K, L) = AK^{\alpha}L^{1-\alpha}$の一人当 りの生産関数$f(k)$を導き出せ。

108. `[Problem ID #108] cycles: Samuelson-Hansen vs. Hicks`
Independently, Samuelson and Hansen each developed an accelerator-multiplier model which can lead to damped damped business cycles (the state variable converges to an equilibrium value). Hicks extended this model to one in which business cycles need not converge, but remain the same size indefinitely.
Samuelson-Hansen はビジネスサイクルモデルを開発し、サイクルが存在するが縮んでいって均衡に収束する。Hicksの延長モデルではサイクルがずっと続いて均衡に収束しない。

The "multiplier" equation is basically the same: $C = cY$. Describe the two differences in the "accelerator" equation, and the difference in the "boundary conditions", between the Samuelson-Hansen model and the Hicks model.

両モデルのマルチプライア式は同じく $C = cY$ だ。モデルのアクセラレーター式の違いを２ヶ所、限界制限を１ヶ所、について述べろ。

109. `[Problem ID #109] OLG: dynamic structure`
Describe the dynamic structure of the overlapping generations model explained in class.
広義で説明された overlapping generations model の動学的構造を述べろ。

(a) What is the *life cycle* of an individual member of a generation?
世代の各個人のライフサイクルを説明せよ。

(b) What is the generational structure of the population in each period?
世代による人口の構造を説明せよ。