Economic Dynamics

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Abstract

In Lecture 7, Part 1, we look at the theory of pure exhaustible resources.

Steady States are Impossible with Pure Exhaustible Resources

- Actually, there is an infinite set of possible steady states, but they are all uninteresting, except for one, and that one is impractical.
- Steady states where the state variable is the stock imply zero consumption. Uninteresting and clearly not economic equilibrium.
- As in growth theory, consider per capita stock as the state variable. Positive consumption requires decreasing stock, so maintaining steady state with positive consumption implies decreasing population.
  - Matching willingness to accept restrictions on family size and the preferred level of resource consumption seems unlikely.
- This implies we need a utility function (criterion for trading consumption in one period against consumption in another).

Intergenerational Considerations

- Since people live a finite time, it is nearly certain that their preferences for consumption during life will differ from their preferences for consumption later, even if they care about their children and future generations.
  - They may value them less, or
  - Care about their (differing) utility rather than specific consumption, or
  - Be imperfectly informed about the future.
- But the current generation's actual choices change the *constraints* for future generations in a way the future generations cannot affect the current generation.
- This is different from growth theory where you can always return to the "golden rule" steady state.
- For simplicity, we ignore issues of intergenerational equity, but they are extremely important in practice (consider Japan right now!)

Exhaustible Resources Compared to Growth Theory: Assumptions

- We consider resources which exist in finite amount, and necessarily are depleted by consumption.
- Contrast growth theory based on factors which are produced and accumulated without bound: capital, technology.
- It's not enough to just change the sign. There is a lower bound of *zero* for exhaustible resources; they cannot keep decreasing at a given rate forever.

Exhaustible Resources Compared to Growth Theory: Modeling

- Because of depletion, we cannot use steady state analysis.
- Steady state analysis, although based on a *dynamic model*, allows us to think about the economics in the same way as we do with equilibrium.
- Without a "good" steady state to aim at (*e.g.*, Solow's Golden Rule), we *must* make explicit intertemporal tradeoffs, *i.e.*, dynamic economic analysis. It's not possible to reduce the (pure) exhaustible resource problem to a pure dynamic model that is economicly interesting.

Why infinite horizon?

- If we set a finite planning horizon T, then a rate of consumption of $1/T$ is an obvious plan, and usually a pretty good approximation to the best plan.
- But this naturally uses up all of the stock.
- This is not a problem in inventory management: you just order more for the next planning period. But the definition of *exhaustible* is that once used up, there will never be any more. For any reasonable finite horizon, the question of "what do we do after the resource is used up?" becomes critical —we, and the need that the resource satisfied, will still exist "after."
- Since it's hard to imagine that if you manage to survive to day N, there is *zero* chance of surviving to day $N + 1$, it becomes natural to consider an infinite horizon (this is the *principle of mathematical induction*).

Infinite Horizon Models

- We prefer models with an infinite horizon because they correspond to an autonomous recursive model. That is, we know that tomorrow is mostly like today.
- In particular, just as tomorrow follows today, the day after tomorrow follows tomorrow. Since tomorrow can never be yesterday, time is an infinite sequence of days. There's always a next day and it is never a day we've already experienced.
- Also, with finite horizon there is the technical problem of "end-point effects": if the calculation ends at time T, then it will give the highest value to consuming all remaining goods at time T. Such behavior is not only unrealistic, the expectation of an "end-time potlatch" alters incentives in the near future.

Preferences in Infinite Horizon Models

- Since human needs change little from time to time, and must be satisfied at each point in time, we use a (additively) *separable representation* of preferences: $U(c_0, c_1, \ldots) = \sum_{t=0}^{n} u_t(c_t)$.
- It seems natural to simplify by assuming symmetry, $u_t(c_t) \equiv u(c_t)$ for all t. But there are two difficulties:
  - In steady state, $\sum \bar{u} = \infty$ if the steady state $\bar{u} > 0$, but maximization is impossible because we can't compare infinities in a useful way! We can compare steady states (as we did in Solow), but we can't be sure that there is no non-steady state preferred to any steady state (as happens in the "collapsing competitive fishery".
  - With exhaustible resources, we eventually use them up, but still

    $$U(1, 0, \ldots) = U(0, 1, 0, \ldots) = U(0, 0, 1, 0, \ldots) = \cdots.$$

    We can't decide when to consume!

Preferences with Infinite Horizon (2)

- Both problems can be solved with *discounting*:

$$U(c_0, c_1, \ldots) = \sum_{t=0}^{\infty} \delta_t u(c_t),$$

  where $\sum_{t=0}^{\infty} \delta_t < \infty$. Usually we put $\delta_t \equiv \delta^t$ (so that $\delta_0 = 1$).

- Other possibilities include
  - *long run average utility*:
    $U(c_0, c_1, \ldots) = \lim_{m\to\infty} \sup_{n>m} \sum_{t=0}^{n} \frac{u(c_t)}{n}$,
  - and the *overtaking criterion*:
    $(c_0, c_1, \ldots) \succ (c'_0, c'_1, \ldots)$ if and only if there exists n such that $c_t > c'_t$ for all $t > n$.
  - These emphasize the very long run, and so are used for evaluating government policy. They're not appropriate for consumer or business optimizations.

Constraints in Infinite Horizon Models

- Constraints are easier than preferences.
- A bound on total consumption of the resource over time: $\sum_{t=0}^{\infty} c_t \leq X_0$, or
- a recursive constraint as in Solow's growth model:
  $K_{t+1} \leq K_t - D_t + F(K_t, L_t)$ (Solow used a differential equation)
- or period by period constraint $c_t \leq \bar{c}_t$ or $c_t \geq \underline{c}_t$.

Oil: A Pure *Exhaustible* Resource

- Oil exists as a stock, and is used up, *i.e.*, "exhausted." Once the stock is exhausted, there will never be any more.
  - This is an approximation: after 20 million years or so, new stocks will form in the ocean floors. But we use up oil *much* faster than that.
- Oil is *storable*, so the rental price (price of consumption) must be equal to the asset price. True of any inventory, but in theory of the firm we equate assets to (generic) capital. (*E.g.*, rental car.)
- Resources like oil that must be used up are called *pure exhaustible resources*.
- Since oil cannot be recovered once used up, several conceptual issues arise:
  - The model should have an infinite horizon.
  - Steady states are impossible.
  - Although this lecture abstracts from them, intergenerational considerations are important.

Pure Exhaustible Resources

- Abbreviate *pure exhaustible resource* to *exhaustible resource.*
- Exhaustible resources are *rival* goods.
- They may be non-excludable (ocean fishing), partially excludable (large oil fields), or completely excludable (small oil fields).
  - Socially optimal usage patterns don't vary; they depend only on the stock.
  - Degree of excludability helps determine market structure, and the equilibrium usage patterns are different.
- The basic ideas of dynamic optimization can be seen with minimum technical difficulty in a model of a monopoly business which owns the whole stock of a resource.

Monopoly

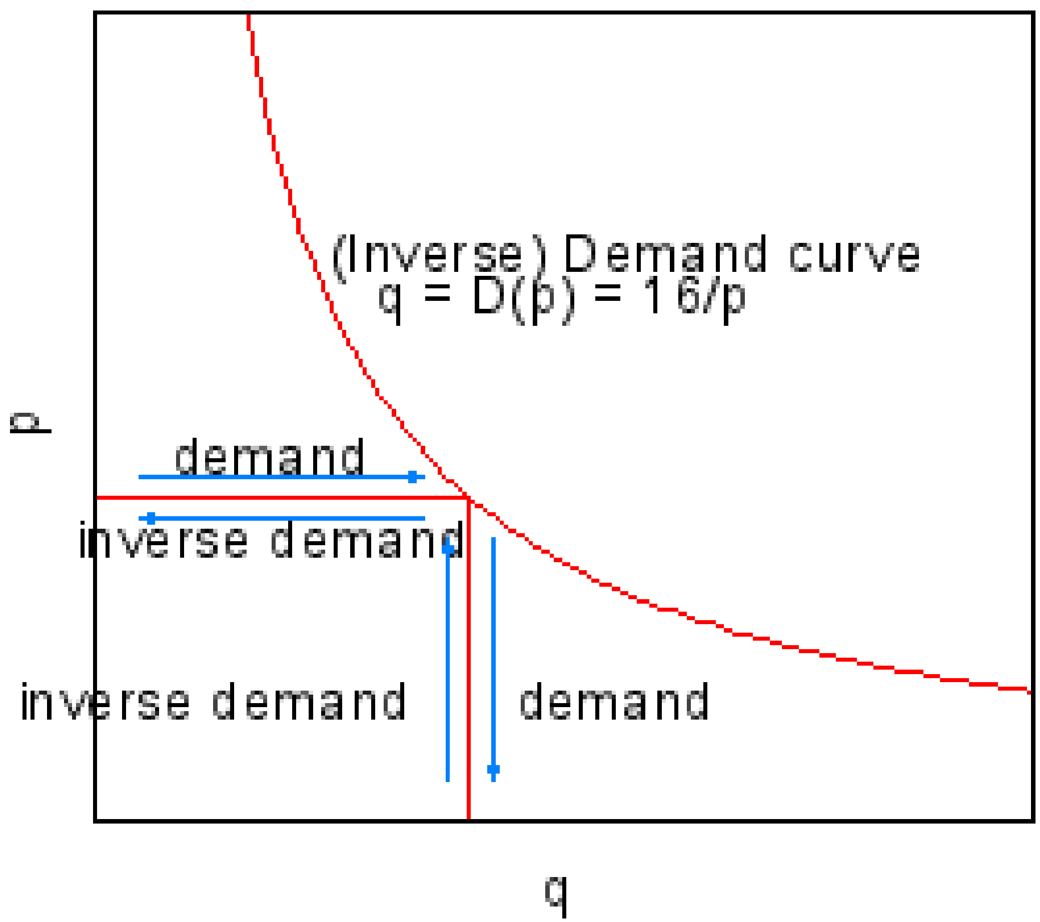
- Single-decision-maker, completely excludable exhaustible resource.
  - Social planner or monopoly; monopoly is simpler.
  - Unlike intellectual property, there are no spillovers from consumption.
  - Monopoly (or government) is special: other producers are excluded.
- Monopoly production subtracts from the *stock* of the resource, but provides a *flow* of benefits to consumers. Like saving in growth theory, where production (Y) can be used as consumption ($(1-s)Y$) or savings (sY), and when saved is *added* to the capital stock ($\dot{K}$).
  - We suppose production is costless for simplicity; marginal analysis is possible as usual.
- Critical point: price of the good sold to consumer must equal price of the good saved as asset: they are perfect substitutes.

Consumer Behavior

- Consumer behavior: market demand curve.
  - Consumers do not store the good.
  - The demand curve will be the same for all market structures.
  - Examples: *linear* (constant-slope) and *constant-elasticity* demand curves.
- Assume the market demand function for the resource is constant over time. At each instant of time, the relation between price and total quantity demanded of the resource is the same.
  - After covering the basic theory, we will consider the adjustments that must take place in a growing economy.

Demand

- We denote the instantaneous or one-period *demand curve* by $q = D(p)$. Recall that we also use the *inverse demand curve* $p = D^{-1}(q)$, which has the same graph. The inverse demand curve interpretation is also called the *marginal willingness to pay* curve.



Dynamics and Demand

- With an exhaustible resource, price must rise to choke off demand. Let current stock be S_0. Suppose $p_t \leq \bar{p}$ for all t. Then $q_t = D(p_t)\bar{q} \equiv D(\bar{p})$ since demand is downward sloping. For example, in the figure above, you could take $\bar{p} = 4$, implying $\bar{q} \equiv D(4) = 4$.
- Stock is exhausted no later than time $T \equiv S_0/\bar{q}$, when price must rise to make quantity demanded equal to 0, to maintain equilibrium.
- Setting price $= \bar{p}$ until stock runs out, then jumping to choke price, is nonsense. The marginal customer at time T has much lower value than the marginal customer at time $T+1$. Under a plan to exhaust the resource in time T, both a profit-making firm and a social planner want to reduce consumption in time T and increase it (to greater than zero) in time $T+1$.
  - This applies to any pair of periods. Price rises gradually, forever.

Asset Pricing as Portfolio Choice

- The exhaustible resource is an asset, and is priced by comparing it to other assets, in particular, bonds. We consider whether the firm's future profitability is increased by selling more of the resource and buying more bonds, or by selling less of the resource and buying less bonds (*N.B.* marginal analysis).

- Suppose that at date 0 the market price of the resource is P_0. This is both the price as a commodity (sold to consumers) and as an asset (held for its future value).
- Suppose that the firm can buy or sell bonds with an interest rate of r. (That is, paying 1 yen for a bond today will yield a return of $1 + r$ yen tomorrow.)

Asset Pricing as Portfolio Choice, cont.

- The firm decides quantity by comparing two plans for the marginal unit:

  a. Invest p_0 in the resource now, returning p_1 tomorrow.

  b. Invest p_0 in bonds now, returning $(1+r)p_0$ tomorrow.

  The investments are the same, so we just compare the future values:

$$(1+r)p_0 > p_1.$$

- Then the firm faces one of three situations about P_1, the next period price:
  - $p_1 < (1+r)p_0$. Bonds increase in value faster. The resource is *overpriced* today.
  - $p_1 = (1+r)p_0$. neither increases in value faster. the resource is *correctly priced* today.
  - $p_1 > (1+r)p_0$. Resource value increases faster. The resource is *underpriced* today.

  In present value terms, $p_0 > \frac{1}{1+r}p_1$.

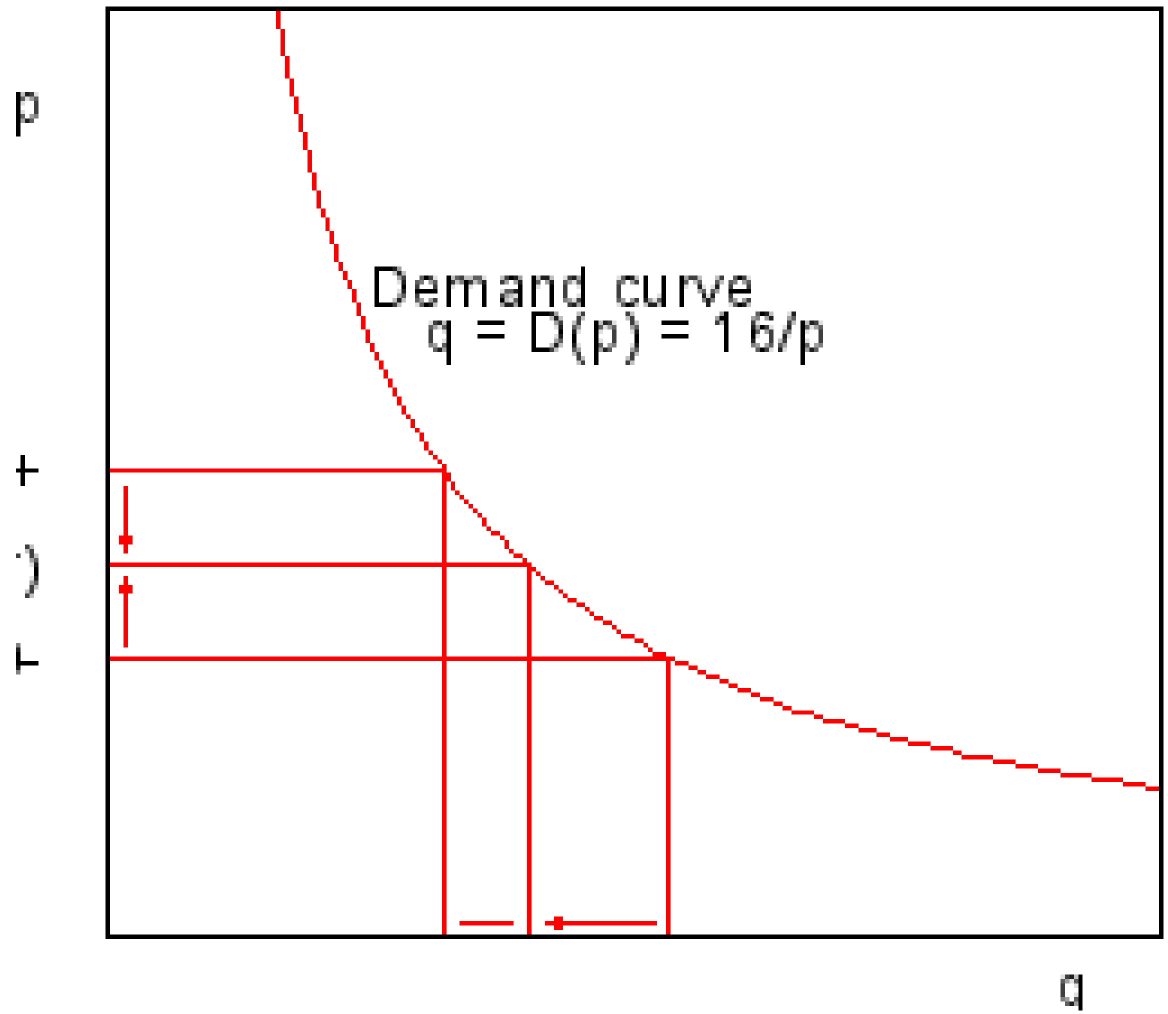
Arbitrage

The preceding argument is called an *arbitrage argument. If* there is a sure way to make money by changing your portfolio of assets in the financial markets, *then the market is not in equilibrium.* Taking advantage of such a situation is called *arbitrage.* The activity of arbtrage tends to have adverse effects on the *terms of trade*, *i.e.*, increasing demand, and price, in the lower-priced market, and decreasing price by increasing supply in the higher-priced market. Eventually the price differential is eliminated, and the markets are in equilibrium.

This gives us an equation, the two markets must have the same price, that characterizes equilibrium. So, we have learned how to evaluate the price path by looking at the *financial market equilibrium.* This is common in dynamic studies.

Interaction of Investment Decision and Market Price

The firm is a monopolist, facing downward sloping demand. Its "investment decision" moves the terms of trade against it.



Dynamic Price Path

The firm must expect the price to rise over time according to the rule $p_{t+1} = (1+r)p_t$, *i.e.* $p_t = (1+r)^t p_0$.

Otherwise the firm will want to "play the market," but this also alters the market price of the exhaustible resource, returning the price to this path.

For given p_0, at time t the depletion is $D((1+r)^t p_0)$, and the stock at future time T is

$$S_T = S_0 - \sum_{t=0}^{T-1} D((1+r)^t P_0).$$

Initial Price p_0

- For given p_0, at time t the depletion is $D((1+r)^t p_0)$, and the stock is

$$S_T = S_0 - \sum_{t=0}^{T-1} D((1+r)^t p_0).$$

- Obviously price P_0 cannot be too low, or $S_0 < \sum_{t=0}^{\infty} D((1+r)^t p_0)$, and the stock is used up in finite time.
  This would drive price to infinity, higher than the path assumed ($(1+r)^t p_0$ is finite).
- What if price p_0 is too high? Then $S_0 - \sum_{t=0}^{\infty} D((1+r)^t p_0) > 0$, and some of the stock is never used. This could happen for inelastic demand.
- Optimal initial price for monopolist depends on elasticity of demand.

Related Results

- If the choke price is not infinite (*e.g.*, with linear demand), in equilibrium the stock will be used up on the date when the price hits the choke price.
  - Interpretation: there is a perfect substitute whose price is the choke price.
  - So when price hits that level switch to the substitute.
- The social planner will surely use up all of the stock at infinity; this results in maximum benefit to society.
- If stocks are excludable, a competitive market in stocks will achieve the social optimum.
- If stocks are not excludable, the tragedy of the commons probably results in overexploitation, and possibly exhaustion in finite time.

The Hotelling Rule

- As usual, it is often convenient to solve a similar model in continuous time.
- By a usual kind of limiting argument applied the length of each period of time in the discrete arbitrage equation:

$$p(t+\delta) = (1+r)^{\delta} p(t),$$

  we can compute a continuous price path according to the differential equation

$$\dot{p} = rp.$$

- This is called the *Hotelling Rule* (after the economist Harold Hotelling).

Initial Condition and Equilibrium

- We have characterized the price path in terms of the no-arbitrage condition (the Hotelling Rule). But we haven't given the level of the price.
- First, integrate Hotelling's Rule to get $p_t = p_0 e^{rt}$ (again assume r constant over time for computation).
- Here, let us consider competitive supply (*e.g.*, in a case with many individual firms, each owning a single small oil field). Now let's try the consumption path where quantity supplied is equal to quantity demanded, at a price determined by inverse demand.
- Then the long-run resource balance condition gives $\int_0^\infty D_t(p_0 e^{rt}) dt = S_0$, which is complicated but can be solved to get p_0^*.
- So we propose a price path of $p^*(t) = p_0^* e^{rt}$.

Verifying the Equilibrium

- Of course "supply = demand" strongly suggests market equilibrium, but we need to confirm that all agents are making optimal decisions.
- Is $p^*(t) = p_0^* e^{rt}$ an equilibrium? Yes:
  - The condition of consumers' optimum in each period is implied by the assumption that price is at the inverse demand of the quantity sold. Such consumers are *myopic* (near-sighted): they don't consider the future.
  - No firm can profit by selling more now: they run out in finite time because of resource balance.)
  - The argument against selling less now is not the "mirror image"! Resource balance is not a constraint against consuming less.

Uniqueness of Equilibrium

- Have we shown that there is *no* equilibrium that leaves some resource "left over at the end of time"?
- No! Because Hotelling's Rule makes them indifferent, the "other firms" *may* optimally choose to fill up the gap left by the leader now, and "give way" to the leader when it comes back in the future, keeping price the same. Then, they are indifferent between earning now and later (when the *leader*, the firm that sells less now, sells its "excess reserve") because of Hotelling's Rule.
- However, there may be other equilibria. It may be that all of the firms decide to "follow the leader" and maintain a higher price. As long as forever after, the price follows Hotelling's Rule from the current price, no firm has an incentive to change its behavior and sell its reserve early, as long as it believes the other firms will *not* "give way" (and therefore the price will drop).

Is There a Proof of Multiple Equilibria?

- Can this argument be proved? The answer is that it depends on the elasticity of demand and the interest rate. If demand is very inelastic and the interest rate high, the profit to reducing supply now (and forcing price up) may be great enough to support a restriction on supply for ever.

- But if demand is elastic, profit from selling a little more now at (almost) no reduction in price is very great. Others will eliminate excess demand, returning price to the level at which the stock is just exhausted "at the end of time."

- Then that equilibrium is unique.

Rational Price Bubbles

- A "bubble" is a price path that is not supported by fundamental value of the good or security. Normally bubbles are *irrational*, and eventually "burst" when it becomes clear to some agents that excess prices cannot be sustained by actual value. However, in this market, when the equilibrium is not unique, the non-competitive (high price) equilibria are what is called "rational bubbles".