Economic Dynamics

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Lecture 7: November 16, 2018

Abstract

We consider the dynamics of competitive fisheries and network industries.

The Competitive Fishery

- Classic example of a dynamic model more complex than Solow growth model and derivatives.

Competitive Analysis in the Fishery

- Competitive market for the harvested good at price q.
- Then q does not depend on t or Z. What happens is that the value of taking the fish out is compared to the value of future harvest; since q doesn't change, the intertemporal comparison reduces to the own rate of return.
- Variable input to harvesting; *e.g.*, ships or boats used for fishing, denoted X.
- Catch per boat depends on Z, size of population, and X, the variable input: $f(Z, X)$.
- Total catch is $F(Z, X) = Xf(Z, X)$.
- Assume $f_Z \geq 0$, $f_X \leq 0$, and assumptions on the macro function are $F_X > 0$, $F_{XX} < 0$, $F_{XZ} > 0$ (fish and chips are complementary), $F(0, X) = F(Z, 0) = 0$ (both fish and ships are essential).
- Hitting the boundaries: in deep-sea fishing, density of fish is constant, and $f_Z = 0$; density of boat might be constant, no crowding, then $f_X = 0$.

The Fishery Model

- Discount is *own rate of return*, based on decision to abstain from consumption
- In $f(Z, X)$, X is not the amount of effort by the boat; it's an externality from other boats.
  - Example of *take-no-ko* at Osaka Shaken. Think of X as non-durable (labor, rental capital goods).
- Of course Z affect this.
- Multiply by X to get industry yield
- Derivative conditions are reasonable, and allow us to prove existence of economically sensible equilibria (correspond to physical feasibility and experience)

Non-Externality Case

- Allow case of no externality of boats, no effect of density
- But normally signed
- Example of non-externality case: open sea fishing. The fish spread out, so having more fish doesn't increase catch. Boats spread out too, so having more boats may not cause crowding.
- Take-no-ko in one takebayashi is crowded, but if there are many forests, no crowding.

Entry Conditions

- Ships are capital, you make an investment, must compare investment in ships to value of *future* catch. Fishermen realistically have a long run perspective.

  E.g., Japanese fishermen. Not good outlook because of competition from low-wage countries—young Japanese don't want to enter.

- But we already have a problem of backward induction, so adding in strategic long run behavior would be very difficult.

- *Myopic behavior* just looks at the current situation; *e.g.*, a differential equation.

- Also contestable markets. Schooling fish—take ships from one school to another. Free entry, no sunk cost *vis-a-vis* a given fishery. Airline example of contestable markets. Many many markets—city-pairs, direction, time of day, day of week, *etc.*.

Firm Objectives and Constraints

- Rental price p for the input, even if it's capital (want to work instantaneously, not with stocks). *E.g.*, taxi drivers.
- $\pi = qf(Z, X) - p$.
- Friction to new entry, $\dot{X} = \mu\pi$, with $\mu < \infty$.
  - Technical aspect: Without friction, instantaneous adjustment leads to discontinuity and thus nondifferentiability.
  - Economic justification: different tastes (risk), expectations, and access to funds for investment among fishermen.

Flow Cost (Rental) on Ships

- Ships are capital, but we will avoid this complication.
- Want to look at *myopic behavior*; use a differential equation depending on a state variable—don't look in the future.
- In take-no-ko example, labor is non-durable.
- We can make capital (ships) non-durable, by considering a rental market.
- Rental price of capital is p. (Or wage to labor.)

Ship (Firm) Profits

- $\pi = qf(X, Z) - p$
- As μ goes to infinity, can show limit is appropriate

A Preview of Dynamic Behavior

- Converge to the maximal catch.
- Converge to some other steady state.
- Decrease without converging, except to zero.

Steady State in the Fishery

- As with the Solow model, we look for a steady state $\dot{X} = 0$ at all t, and then $\pi = 0$, so we have $p = qf(Z, X)$ at all t.
- Then we have the other condition, which is $\dot{Z} = H(Z) - F(Z, X)$.
- Need an Inada condition for sufficiently high levels of f_X.

Infinite Speed of Adjustment, cont.

- Convergence speed infinite implies immediate convergence to long run equilibrium.
- So at all times $p = qf(Z, X)$.
- This is a necessary condition, so if the limiting argument works, it must give the same. The limiting argument gives sufficient condition, but it's too difficult here.
- Thus price satisfies the equation above because of zero-profit condition.

Resource Balance Condition

- Then $\dot{Z} = H(Z) - F(Z, X)$.

An Example Yield Function

- Cobb-Douglas format: $f(Z, X) = Z^a X^b$
- Solution using zero profit condition: $X_t = \left(\frac{q}{p}\right)^{\frac{b}{1-b}} Z_t^{\frac{a}{1-b}}$
- We don't know if this is concave or convex.

Conditions on Z_0

- Since economic activity by humans is relatively recent, probably the Z_0 was near stable steady state with $Y = 0$.
- Then time path is decreasing to new stable steady state.

Projecting the Terminal State

- Note that again monotonic decreasing with time.
- It's hard to distinguish between collapsing fishery and fishery decreasing to steady state.
- Empirically similar:
  - Prices exogenous, nothing useful about state of fishery here.
  - Can get numbers on inputs and catch.
  - If this were a farm, we'd be able to count the population, but this isn't a farm. To estimate the population, we need to use a quantitative model (like the one we're using here).
- Let $\hat{Z}$ maximize $H(Z)$. But Z^* and $\hat{Z}$ have no logical relationship.

Entry Conditions

- Ships are capital, you make an investment, must compare investment in ships to value of *future* catch. Fishermen realistically has a long run perspective. *E.g.*, Japanese fishermen. Not good outlook because of competition from low-wage countries—young Japanese don't want to enter.
- But we already have a problem of backward induction, so adding in strategic longrun behavior would be very difficult.
- *Myopic behavior* just looks at the current situation; *e.g.*, a differential equation.
- Example of *take-no-ko* at Osaka Shaken. Think of X as non-durable (labor, rental capital goods).
- Also contestable markets. Schooling fish—take ships from one school to another. Free entry, no sunk cost *vis-a-vis* a given fishery. Airline example of contestable markets. Many many markets—city-pairs, direction, time of day, day of week, *etc.*.

Cobb-Douglas Example

- Cobb-Douglas format: $F(Z,X) = Z^a X^b$, $0 < a, b < 1$.
- Then $f(Z,X) = Z^a X^{b-1}$.
- Solution from zero-profit condition $p = qf(Z,X)$:

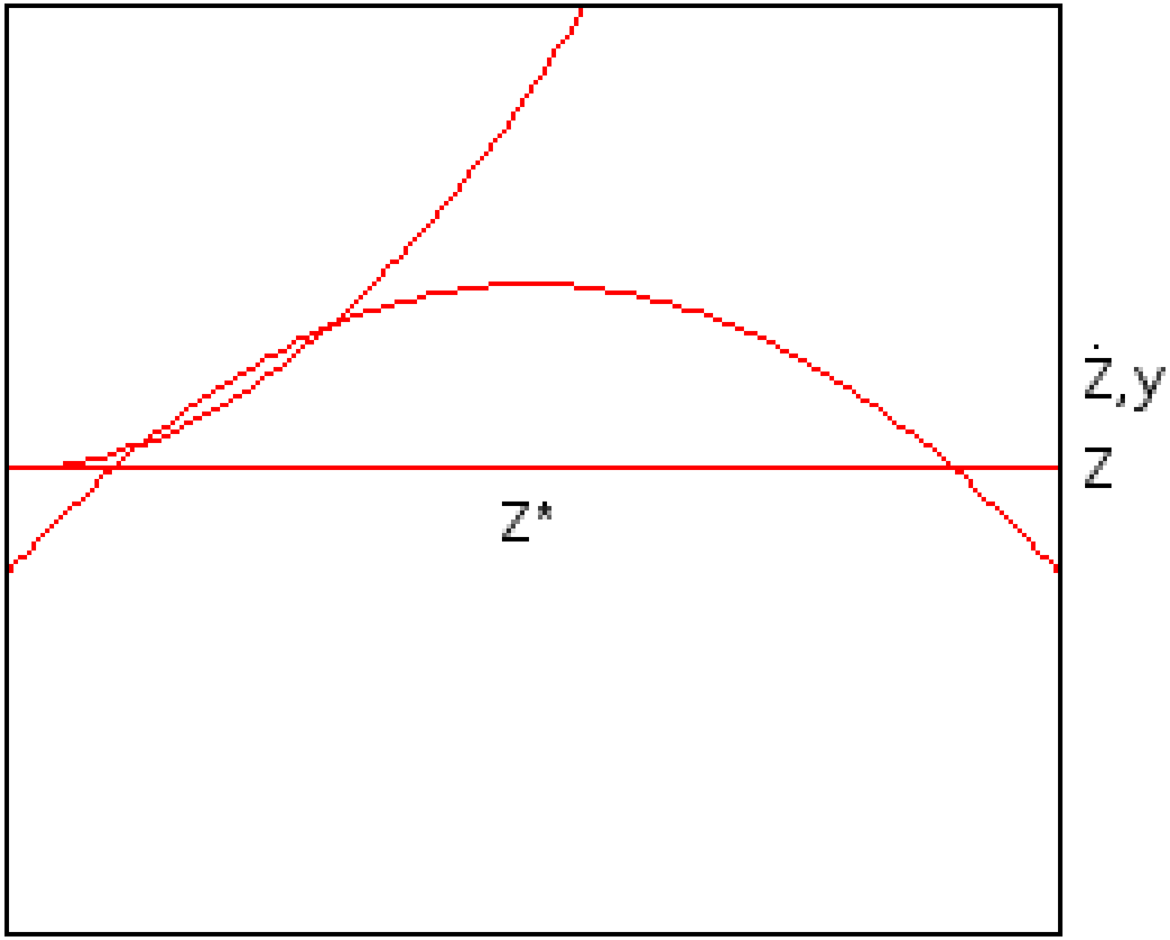
$$X = \left(\frac{q}{p}\right)^{\frac{1}{1-b}} Z^{\frac{a}{1-b}}.$$

- Differential equation $\dot{Z} = H(Z) - F(Z,X)$
- Substituting for X, $\dot{Z} = H(Z) - \left(\frac{q}{p}\right)^{\frac{b}{1-b}} Z^{\frac{a}{1-b}}$.
- This gives a form with the shape of H and the second term is a positive, increasing function, though we can't say if it's concave or convex.
- Let $A = \left(\frac{q}{p}\right)^{\frac{b}{1-b}}$ and $\beta = \frac{a}{1-b}$.

Solutions

- With H bell-shaped,
- Two possibilities
  - Case 1: intersects H. Then there are two intersections, lower unstable, upper stable, focus on stable case.
  - Expect to start from Z_0, so population starts high, monotonically decreases over time to new $\bar{Z}$. (In any case if $Z_0 > \underline{Z}$, converge to $\bar{Z}$.)
  - Case 2: q and p exogenous, easy to imagine q large, giving a picture with the one curve everywhere above H, and collapse to $Z = 0$, regardless of Z_0.
- Suppose q and p endogenous. This doesn't help in the collapse case; they go in the wrong directions as the fish population decreases.

Phase Diagram for Fishery: Stable



A stable competitive fishery

Conditions on Z_0

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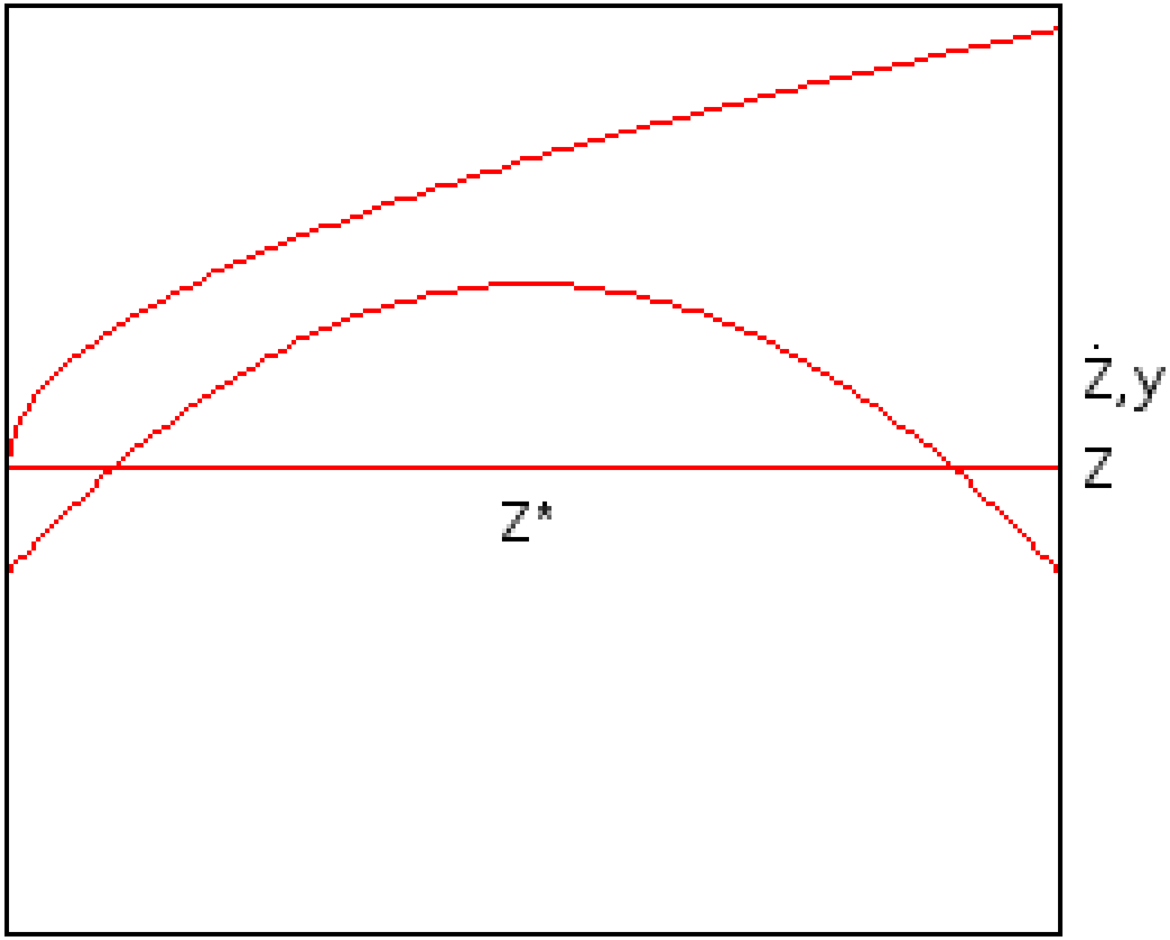
Phase Diagram for Fishery: Collapse



A competitive fishery that collapses.

Projecting the Terminal State

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Fishery Management

- The decision y_t is the *yield* or "catch": how many tons of fish to take from a *fishery*, an area of ocean where fish live. The utility is $U_t(y_t) = \ln y_t$.
- The state variable is the *stock* of fish in tons, X_t, which follows the law of motion $X_{t+1} - X_t = 0.05X_t(100 - X_t)/50 - y_{t+1}$.
- The value function is $V_t(X_{t-1}) = \max_{y_t}(\ln y_t + 0.98V_{t+1}(X_t))$. 0.98 is a *discount factor* saying that the future is 0.98 times as valued as the present.

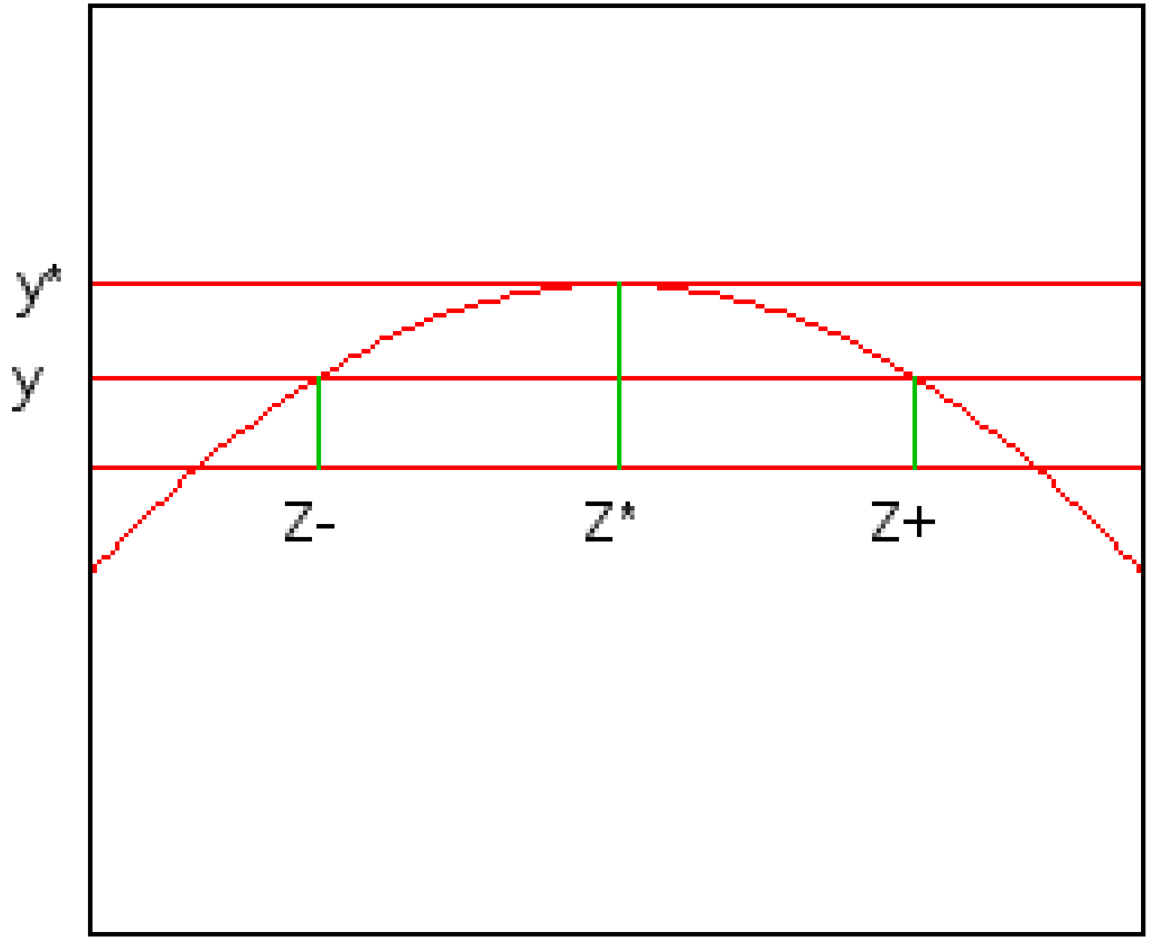
Steady State Optima in the Fishery

- Instead of initial and terminal conditions, we look for a *steady state*, or a sustainable catch. Then we suppose $V_t(X) = V(X)$, $X_t = X$, and $y_t = y$ for all t.
- The law of motion is $\Delta(X, y) = 0.001X(100 - X) - y$.
- The steady state condition is $\Delta(X, y) = 0$.
- You might think the value function becomes $V(X) = \max_y(\ln y + 0.98V(X))$ ...
- and it is natural to solve the value function $V(X) = 50 \ln y$, then substitute for y using the *steady-state* law of motion, and finally maximize over X, but *this is incorrect.*

- We must solve the maximization problem *without* the steady state condition on variables (but we need the steady state condition on the *value function*). Typically there are many solutions, we need to find one which is a steady state.
- The value function is $V(X) = \max_y(\ln y + 0.98V(X + \Delta(X, y)))$.

Constant Harvest Rate

Suppose the population is harvested as rate Y_t. Consider the stationary policy $Y_t = Y \leq \max H$ for all t. Note that there are two steady states, unless $Y = \max H$, when the unique steady state is *unstable*.



Harvest Policy in a Feedback Loop

- Use the policy $Y_t = Y^* + \zeta(Z_t - Z^*)$ to achieve a stable maximum harvest steady state.

Regulation of the Competitive Fishery

- Self-regulation or regulation from the government. *E.g.*, a sympathetic view of *dango*.
- Sometimes it works, but often it seems to fail. Why?
- Measuring population is hard; measuring catch is easy, but population estimates depend on imputation via a model of the catch process. (*E.g.*, the model we're using!) If you could observe the stock, you'd be farming.
- We don't know much about population dynamics (the shape of H). Better knowledge of plants and animals than rational, strategic humans, but still poor knowledge.
- Now theory makes it likely in both Case 1 and Case 2 that things will decrease over time. But you can't even know if you've passed $\underline{Z}$ (the point of no return).
  - *E.g.*, the anti-whaling controversy. Still don't really know how close to $\underline{Z}$ the great whales got.

Regulating the Fishery

- Can control catch, or entry by boats.
- Despite the fact that the open sea is hard to regulate, the product has to come to market, people need to know where to go (advertisement?)—the government will find the sellers or at least the buyers. Maybe indirect, but regulation is possible.
- Assume direct regulation.
- Let the goal be maximizing present value of catch.
- First consider case of costless catch. Then we just maximize present discounted sum of revenue:

$$\int_0^\infty e^{-rt} q Y_t dt.$$

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Optimal Regulatory Policy

- May be reasonable to assume government maximizes industry surplus, approximately the same as profit.
- Maximize present value of surplus.
- Simple case: government is in perfect control. Take economic value of harvest and time in account.
- Assume a costless harvest.
- So simply maximize the value of the catch, discounted at some rate. If the price is exogenous and constant, it won't affect anything, so set it to 1.
- Then value of capital gives $H'(\tilde{Z}) = r$ at the optimum.

Results

- Suppose that $\tilde{Z} < \underline{Z}$. Then we can see that you wish to use up all the stock as quickly as possible. There is insufficient productivity to preserve a positive stock. (Absent "green" interest groups.)
- Let $\hat{Z}$ be the level of stock that maximizes growth rate of population.
- Note that $\tilde{Z} < \hat{Z}$. From the previous point, $\tilde{Z} < \underline{Z}$ is uninteresting, so assume $\underline{Z} < Z_0 < \tilde{Z} < \hat{Z}$. Now what?
- $H'(Z_0) > r$, so abstaining from harvest makes sense (own rate of return is higher than investing in bond market). But this is true until $H'(Z_0) = r$, so it is desirable to have $y = 0$ until $Z = \tilde{Z}$, then stay there forever, with $y = H(\tilde{Z})$.

The Optimal Policy

- If $Z_0 < \underline{Z}$, the species is doomed, and it makes sense to harvest as fast as possible.
- One important boundary is at $\tilde{Z}$ defined by $H'(\tilde{Z}) = r$. Since $r > 0$, $\tilde{Z} < \hat{Z}$.
- Suppose $\underline{Z} < Z < \tilde{Z}$. By concavity, $H'(Z) > H'(\tilde{Z}) = r$. For a short period θ, clearly it makes sense to leave the fish in the water, let them reproduce, and get $\theta H'(Z) > r\theta$ at the end of the period (the *arbitrage argument*). For $Z > \tilde{Z}$, the arbitrage argument says to
- At $\tilde{Z}$, the government is indifferent, so the optimal stationary policy is $Y = H(\tilde{Z})$ with the population maintained at $\tilde{Z}$.

Dynamic Policy

- If $\tilde{Z} < \underline{Z}$, the government is sufficiently impatient to drive the population to zero in finite time.
- Suppose $\underline{Z} < Z_0 < \tilde{Z}$. The arbitrage argument says to abstain completely from harvesting for a short period.
- This is obviously true all the way up to the point where $Z = \tilde{Z}$.
- Similarly, with $Z_0 > \tilde{Z}$, the government wants to harvest as fast as possible.

Optimal Dynamic Policy

- Thus the optimum policy is
  - If $Z_0 \leq \underline{Z}$, $\tilde{Z} < \underline{Z}$ or $Z_0 > \tilde{Z}$, harvest as fast as possible, stopping when you hit 0 or $\tilde{Z}$, whichever comes first. (We don't know what "as fast as possible" means in this model.)
  - If $\underline{Z} < Z_0 < \tilde{Z}$, harvest nothing and increase the population as fast as possible up to $\tilde{Z}$.
  - If $Z = \tilde{Z}$, harvest $Y = H(\tilde{Z})$, and maintain the optimal steady state.

If Discounting, Why Stationary State?

- Discounting means you want to consume more quickly; how can this be compatible with a steady state?
- Discounting *is* incompatible with a steady state at $\hat{Z}$. Then you do want to trade consumption later for more consumption now at the margin. This is accomplished by consuming extra now, decreasing the stock, which implies inability to maintain that level of consumption in the future.
- But at $\tilde{Z}$, you don't trade one for one. By consuming extra now, you lose $1 + H'(\tilde{Z})$ of future consumption because you're decreasing future stock faster than one for one. This balances out the impatience, leading to optimality of the steady state.

The Whaling Controversy

- The blue whales were hunted to near extinction (in the conservationist sense) by the early 1960s.
- We have a lot of data (appended) from the Committee for Whaling Statistics of Norway.
- Looking at the data we see that efficiency of catch was greatest in the 1930s, by the late 1950s was much less efficient.
- We can also see that the numbers are hardly smoothly varying, it's hard to see a clear trend, other than the average productivity.

The Whaling Controversy, cont.

- So we see that the Japanese, Icelanders, and Norwegians argued that $Z_0 \geq \tilde{Z} > \underline{Z}$.
- The conservationists (and the non-whaling nations) worried that $Z_0 < \underline{Z}$, and/or $F(Z, X^*(Z)) > H(Z)$ for all Z.
- Theoretically, it's hard to say; we look at an empirical model.

An Empirical Model

- Using a discrete model, we have
  - $Z_{t+1} - Z_t = H(Z_t) = Z_t^{\alpha}(A - Z_t^{1-\alpha})$, and
  - $Y_t = F(Z_t, X_t) = AZ_t^{\alpha}(1 - e^{-\nu X_t})$.
- Based on the data, we can estimate this system of equations (done by Michael Spence). We get

$$\nu = 0.0019, \quad \alpha 0.8204, \quad A = 8.356.$$

- Maximum sustainable catch $H(\hat{Z}) = 9890$, while $\hat{Z} = 45177$.

Are the Whales Endangered?

- Using $Y_{1960} = 1987$ and $X_{1960} = 418$, we get an estimate of $Z_{1960} = 1639$. That is, at that pace the whales would be extinct in the end of 1961! (Estimating for 1955-1959 gives 1636, 1496, 1651, 1105, and 1174 respectively.)
- These numbers are pretty suspicious: they claim that for five years running whalers took more whales than were in the ocean.
- The implied natural rates of increase are pretty impressive, though: 1983, 1867, 1995, 1518, 1582, and 1985. Note that these track the catch numbers quite closely. This is not surprising: the model is tuned that way.

Optimal Policy

- Our model implies that the whales are not below $\underline{Z}$, but well below Z^*.
- In fact, the calculation of the optimal policy suggests (using the figures above and $r = 0.05$) a period of abstaining for 9 years.
- $Z^* = 67000$, and $H(Z^*) = 9000$.
- History showed that political aspects are extremely important.

Whaling Data (pre-WWII)

Year	Boats	Catch	Year	Boats	Catch	Year	Boats	Catch
1909	149	316	1920	112	2987	1931	100	6705
1910	178	704	1921	142	5275	1932	186	19067
1911	251	1739	1922	174	6869	1933	199	17486
1912	246	2417	1923	194	4845	1934	242	16384
1913	254	2968	1924	234	7548	1935	312	18108
1914	182	4527	1925	235	7229	1936	254	14636
1915	151	5302	1926	233	8722	1937	357	15035
1916	94	4351	1927	222	9676	1938	362	14152
1917	130	2502	1928	242	13792			
1918	141	1993	1929	337	18755			
1919	154	2274	1930	280	26649			

Whaling Data (post-WWII)

Year	Boats	Catch	Year	Boats	Catch
1945	158	3675	1953	368	3009
1946	246	9302	1954	386	2495
1947	307	7157	1955	419	1987
1948	348	7781	1956	395	1775
1949	382	6313	1957	417	1995
1950	468	7278	1958	420	1442
1951	430	5436	1959	399	1465
1952	379	4218	1960	418	1987

Conclusions

- Profit maximization is consistent with some degree of conservation; it does not imply extinction of the species. Population will be smaller than $\bar{Z}$, but greater than 0 in the steady state.
- If population is depleted below the optimal stationary policy, a profit-maximizing industry would advocate a total ban on harvesting, just like the conservationists!
- The maximum sustainable rate is not a focal point for policy.
- If $\tilde{Z} < \underline{Z}$, profit maximization conflicts with conservation.
- Also, things may be somewhat harsher than it seems here: in the long run, as supply of output falls, you might expect that q would rise, causing harvest rates to rise.
- $\tilde{Z} < \hat{Z}$.

Homework 8: due 2018-11-23, 11:00am

Recall Michael Spence's model of the blue whale fishery. It is a discrete model, where Z denotes population and X the number of whaling ships, and

- The natural rate of increase is defined

  $$Z_{t+1} - Z_t = H(Z_t) = Z_t^{\alpha}(A - Z_t^{1-\alpha})$$

  ,

- and the industry catch function is

  $$Y_t = F(Z_t, X_t) = AZ_t^{\alpha}(1 - e^{-\nu X_t}).$$

Spence estimated the parameters of this system to be

$$\nu = 0.0019, \quad \alpha = 0.8204, \quad A = 8.356.$$

1. Spence calculated that the maximum sustainable catch is $H(\hat{Z}) = 9890$, where $\hat{Z} = 45177$. Verify these calculations.
2. What is the stable steady state population of whales $\bar{Z}$ in this model, assuming catch is $Y = 0$?

3. What is the "point of no return", assuming catch is $Y = 0$?
4. What is the "point of no return", assuming catch is $Y = 1500$?

Network Industries

This section loosely follows Shy, *The Economics of Network Industries.*

- A *network industry* is one which maintains connections among its clients.
  - A market can be thought of as such a service in pure form, allowing its members to compare prices and arrange trades.
  - Most networks are impure, providing connection plus other services.
- Transportation and communication services may be used or not, along with the conceptual connection.
- A software application's file format may be used by a lone user purely to store information, as well as permitting file sharing among users of the same software.
  - Any standard, whether "official" or simply popular, has the same effect of creating a network.
- Networks create markets.

Network Externalities vs. IRTS in Production

- IRTS in production implies that a single large producer is most efficient, by definition. However, with network externalities in consumption, it is both theoretically possible and seen in practice that several providers share a single network.
- A fixed cost with constant marginal cost implies unbounded increasing returns. The model that leads to Metcalfe's law is far less plausible.
- However, the marginal benefit to a network externality is unbounded above, while the reduction in cost due to production ITRS is bounded below by marginal cost.
  - Thus, if network externalities are even slightly increasing in the size of the market, they can be enormous, and may support much larger firms as "minimum efficient size" compared to production IRTS.

Metcalfe's Law

- To the extent that a network merely *provides connections* between users, its value to each user i depends on the set of connections available. We simplify to assuming that it is not the particular set, but rather the size of the set that matters:

$$U_i = u_i(N), u_i'(N) > 0, i \in N.$$

  If u_i is nonlinear, we say *network externalities* are present.

- The simplest estimate of the *value of the network* assumes
  - users are symmetric: $u_i(N) = u(N)$
  - users do not discriminate: $u(N) = u(n)$, where $n = |N|$
  - values are additive: $V = \sum_{i \in N} u(N) = nu(n)$
  - individual value is linear: $u(n) = vn$

- *Metcalfe's Law* is immediate:

$$V = vn^2.$$

A Simple Model with a Network Externality

- We assume a potential market of users M, with $|M| = m$.
- The network externality follows Metcalfe's Law:

$$V = n(nv - c),$$

  where V is the total surplus of the industry, n is the number of users connected to the network, v is the value per connection to each user, and there is a cost of c to stay connected to the network.
  - Unlike the usual theory of the firm, there is a dramatic difference between $c = 0$ and $c > 0$ cases.
- The externality is represented by the coefficient n on v (inside the parentheses).

The Initial Coordination Problem

- Consider the inequality

$$u(n) - c = vn - c < 0,$$

which is the condition where a potential user does not want to join the network.

- It's easy to solve for n:

$$n < \frac{c}{v}.$$

- When $c > 0$ and $v > 0$ is small enough, there may be sizeable populations $n > 0$ such that $u(n) - c < 0$, so the market may fail unless at least $\frac{c}{v}$ users can be convinced to join at the beginning.

- If the initial size of the network is at least $\frac{c}{v}$, the dynamics of the network are qualitatively similar for $c > 0$ and $c = 0$.

Industry Dynamics with Network Externalities

- More interesting than the *fact* that there are increasing returns to size of the market on the demand side is the *effect* of these returns on the dynamics of the industry.

- For example, many innovations start with a single inventor, and as others realize that the innovation is useful, it propagates (or diffuses) through the industry (or even the economy as a whole).

  But with a *pure network good* (one which only offers value by connecting to others) there may be a *minimum viable scale* below which the cost of production is not balanced by the value, even though a large network might have very high net value to each user.

- This means that starting the network requires *coordination* (enough users joining the network at introduction), and therefore the normal market mechanism can fail to support the innovation.

Diffusion Dynamics for a Network Good

- We model the dynamics as a differential equation. The *hazard rate* for joining the network is proportional to the net value to the new user: $\alpha(vn - c)$.
- With m the total population of potential users, multiplying by the *non*user population $m - n$ gives the rate of diffusion:

$$\frac{dn}{dt} = \alpha(vn - c)(m - n),$$

which has the solution

$$n(t) = \frac{m - \frac{c}{v}}{1 + e^{-\alpha v(m - \frac{c}{v})(t - t_0)}}.$$

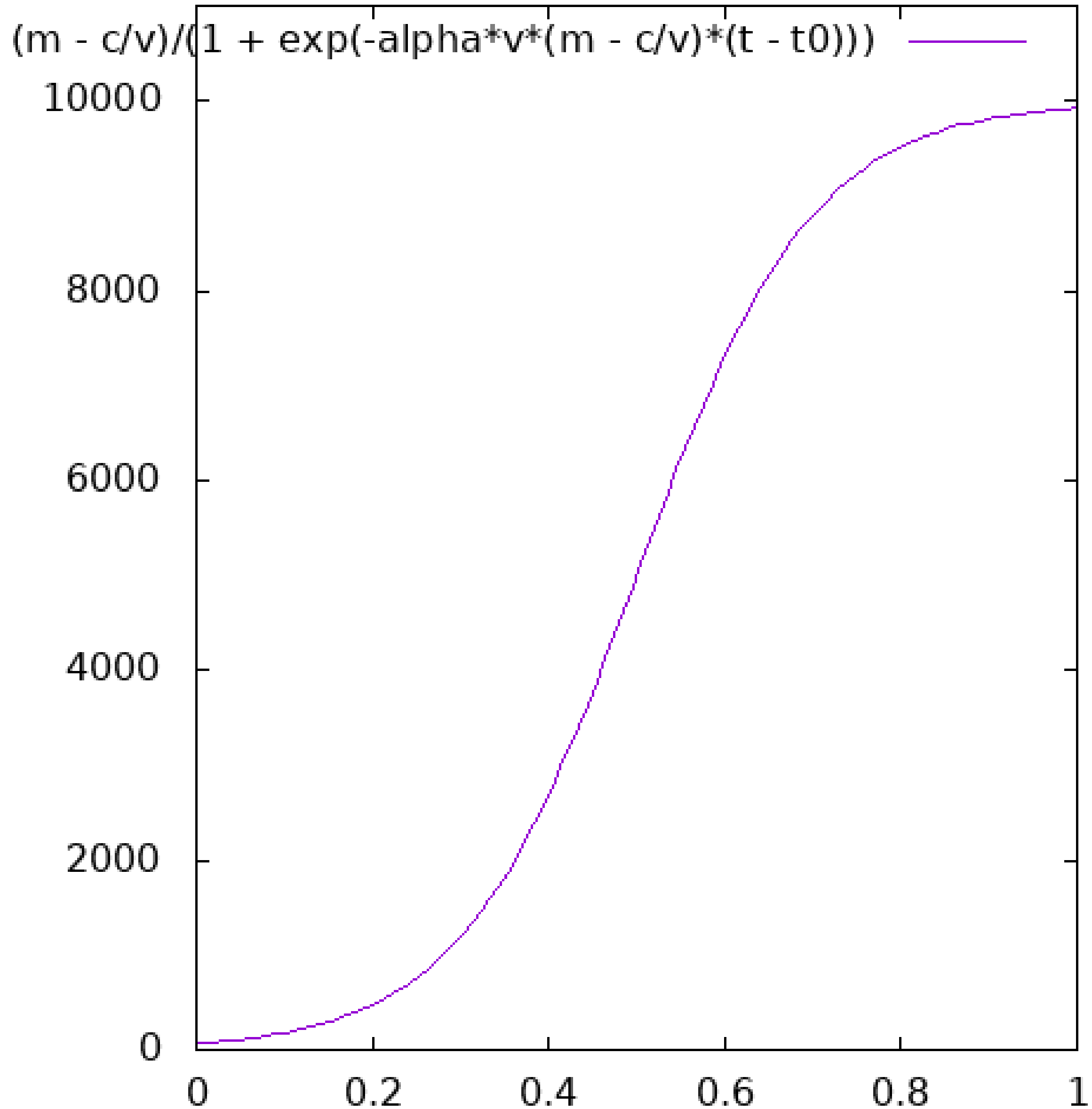
- In the special case of $c = 0$, we can rearrange to get

$$\frac{dn}{dt} = (\alpha v)n(m - n),$$

which is the familar logistic model with solution

$$n(t) = \frac{m}{1 - e^{-\alpha m v t}}.$$

The Logistic Growth Path



The S-shaped logistic growth path is bounded above and below.

Dynamic Competition between Incompatible Networks

- We consider the duopoly, but the principle applies to industries with more than two firms. We have a total population of potential users of m.
- Let the per user per connection values be $v_1 = v_2 = v$, the cost per connection be $p_1 = p_2 = c$, and the number of users (connections) for the two firms be n_1 and n_2.
  - The notations v_i and p_i ("p" for "price") indicate that in a more sophisticated model these might be differentiated or even strategic variables (especially p_i).
- *Incompatible* means that users on one network are *not* connected to the other. Thus for each user, the value of their network is $u_0 = 0$ if not connected to either, and $u_i(n_i) = v_i n_i - p_i$ if connected to network i.

Adoption Decisions of Users

- We suppose that the diffusion of non-users into each network is proportional to net value as in the monopoly case: $\alpha_i(v_i n_i - p_i)$. Once again we will assume symmetry: $\alpha_1 = \alpha_2 = \alpha$.
  - This assumption is more plausible than the assumptions for the "strategic" variables.
- We assume no switching cost, and that existing users switch from 2 to 1 according to the difference in net values: $\delta((v_1 n_1 - p_1) - (v_2 n_2 - p_2))$.
  - Note this hazard rate may be negative.
  - If you were wondering why the hazard rates for non-users have the same α, this switching can help justify that assumption.
  - In a course in economic dynamics, you'd be asked to show when the model with $\alpha_1 = \alpha_2$ and a high δ is equivalent to $\alpha_1 \neq \alpha_2$ and a lower δ.

The Diffusion Model

- Make all symmetry assumptions, and $n_1 > n_2$ at the start of time.
- Then we have

$$\begin{aligned} \dot{n}_1 &= \alpha(vn_1 - c)(m - n_1 - n_2) + \delta v(n_1 - n_2)n_2 \\ \dot{n}_2 &= \alpha(vn_2 - c)(m - n_1 - n_2) - \delta v(n_1 - n_2)n_2 \end{aligned}$$

  Conceptually there are also terms $\pm\delta v \max\{n_2 - n_1, 0\}n_1$ in each differential equation, but on the assumption $n_1 > n_2$, they are zero. On that assumption, we can omit the max in the equations above.

- It is easy to see that $\dot{n}_1 > \dot{n}_2$, and for small enough c, $\dot{n}_1 > 0$. (The last is non-trivial to prove because in the limit non-users and n_2 go to 0.)
- Thus $\frac{d}{dt}(n_1 - n_2) > 0$. $\frac{d}{dt}(m - n_1 - n_2) < 0$ if $\dot{n}_2 \geq 0$, so eventually $\dot{n}_2 < 0$.
- Even with $\dot{n}_2 < 0$, $|\dot{n}_1| > |\dot{n}_2|$, so $\frac{d}{dt}(m - n_1 - n_2) < 0$. $n_2 \to 0$ and $n_1 \to m$.
- Symmetry implies that the opposite conclusions hold if $n_2 > n_1$, so this model is "tippy": whichever network starts out ahead soon crowds out the other.

Dynamic Games

- Mathematical analysis of even the simplest game is quite complex. It's easy to see that if the symmetric model is extended so that each firm can choose price p_i, the firm that starts with greater n_i has a big advantage.
  - As long as that firm is willing to match $p_i = p_j$, it will win the whole market.
  - If the monopoly is expected to continue for a long time, firms may even be willing to offer negative prices.
- If everything is symmetric, the game is very similar to the "War of Attrition", which is known to have only mixed strategy equilibria.

Compatible Networks

- As mentioned, for many networks an *interconnection standard* can be created. This means that (subject to quality of service considerations for the "foreign" users) the network externality is based on the sum of users of all networks in the "internet."
  - Large networks don't have a competitive advantage: several networks of different sizes can share the market.
  - Market structure (number of companies) is more stable.
  - The value to each user is greater (approximately double in the duopoly) so price increase may be more than enough to compensate the leader for allowing interconnection.
- Examples: "The" Internet, protocols such as the "World Wide Web," standards like the "DOM" for web browsers (allows Javascript to work on different browsers) and "ODF" for office automation
- Competition on price and service quality

Standards and "Open Source"

- "Open" standards (no royalty to implement) lead to "open source" implementations
  - "Poor" or hobbyist programmers write their own implementations and contribute them
  - Business customers trying to avoid "lock-in" may write their own implementations and contribute them when they are not mission-critical or competitive advantage
  - Open source businesses may implement to support a further value-added product or service