Economic Dynamics

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Abstract

Last lecture presented Robert Solow's seminal growth model, including the solution, comparative static and dynamic analysis.
This lecture considers a model of a *renewable resource*, whose mathematical expression is quite similar to Solow's model, but whose interpretation is quite different.

Optimization in Dynamic Models

- Solow's basic *growth model* is entirely dynamic. No optimization is present at all.
- The "Golden Rule" is determined by using steady state to reduce the growth model to a single period (although it is repeated indefinitely), and then treating s as a choice variable in a *static* optimization.
- Now we look at *exhaustible resources*, both
  - *renewable resources* (concentrating on the *fishery*), and then
  - *pure exhaustible resources* (such as oil),

  as first examples where economic considerations (optimization and equilibrium) enter.

Pure Exhaustible Resources and Renewable Resources

- Exhaustible resources come in two varieties: *pure exhaustible resources* like oil, and *renewable resources* like fish. *Renewable resources* are "self-renewing" in that if left alone they will grow back to the original stock. However, like pure exhaustible resources (and unlike capital) there is an upper bound on the stock.
- Biological resources like fish are also similar to pure exhaustible resources in that once exhausted, there will never be any more (Jurassic Park excluded).
- Exhaustible resources are necessarily *storable*. Such resources have the property that the rental price (price of consumption) is equal to the asset price. (Compare: is the price of a rental car equal to purchase?)
- You might think that a pure exhaustible resource is just a special case of renewable resource, with the *natural rate of increase* set to zero. In some ways this is true, but each type has its own natural mode of analysis.

Summary: Exhaustible Resource Definition

- An *exhaustible resource* is a good where the stock is bounded and the natural rate of increase has an upper bound, but can be consumed arbitrarily quickly. If completely depleted, no more will ever exist.
  - A *pure exhaustible resource* cannot increase at all. Once any portion is consumed, that quantity is gone forever.
  - A *renewable resource* is an economic good whose stock automatically renews itself at some rate, but this rate (and the stock itself) has some upper bound.
- Renewable resources would better called *self-renewing* resources, but *renewable* is traditional usage and can't really be changed now.

Examples of Exhaustible Resources

- A *pure exhaustible resource* can only be used up; it cannot increase. Oil (or any other mineral resource) is a good, and very important, example of a *pure* exhaustible resource.
- A *renewable resource* is one which has a positive *rate of natural increase*, for at least some level of the stock. Tuna fish (or any other wild biological resource) is an example.
- Solar energy is often called a "renewable energy source." That is reasonable English usage, but solar energy is not a *renewable resource* in the sense used here.
  - The Sun will always be there again tomorrow (no exhaustion).
  - It provides more energy than we can imagine using in the foreseeable future if only we could capture it (effectively unbounded).
- A habitat's "ability to absorb pollutants" may be analyzed as a renewable resource. Consider how Lake Erie (U.S.) or Lake Kasumigaura managed to recover from heavy pollution.

Introduction to Renewable Resources

- Dasgupta and Heal (*Economic Theory and Exhaustible Resources*, 1979) describe these as "resources that are at the same time self-renewable and *in principle* exhaustible."
- Not mineral resources, called "exhaustible," whose stock cannot be increased, although more efficient technology (*e.g.*, deeper oil wells) and discoveries of new stocks may arise.
- Not durable commodities. If the entire stock of automobiles were destroyed, we could still build more and replace them. This might depend on available exhaustible resources, but if the resources were available, the automobiles could be replaced.
- Populations of biological creatures, and agricultural land, are examples. Self-cleansing of polluted water is an example.

Populations

- We borrow terms and ideas from biology, but the exact definitions used often differ.
- A *population* is a group of individuals of the same kind which reproduce themselves, perhaps only in the context of the group. The *population size* is the number of individuals. (In biology, it is often the *total mass*, because as resources become strained individual size decreases. We abstract from this issue.)
- In nature populations tend to increase over time. This increase should depend on
  - the period of time θ over which increase is measured
  - the current population Z_t
  - other factors ζ_t

  giving $Z_{t+\theta} = G(Z_t, \zeta_t, \theta)$.

Typical Laws of Increase

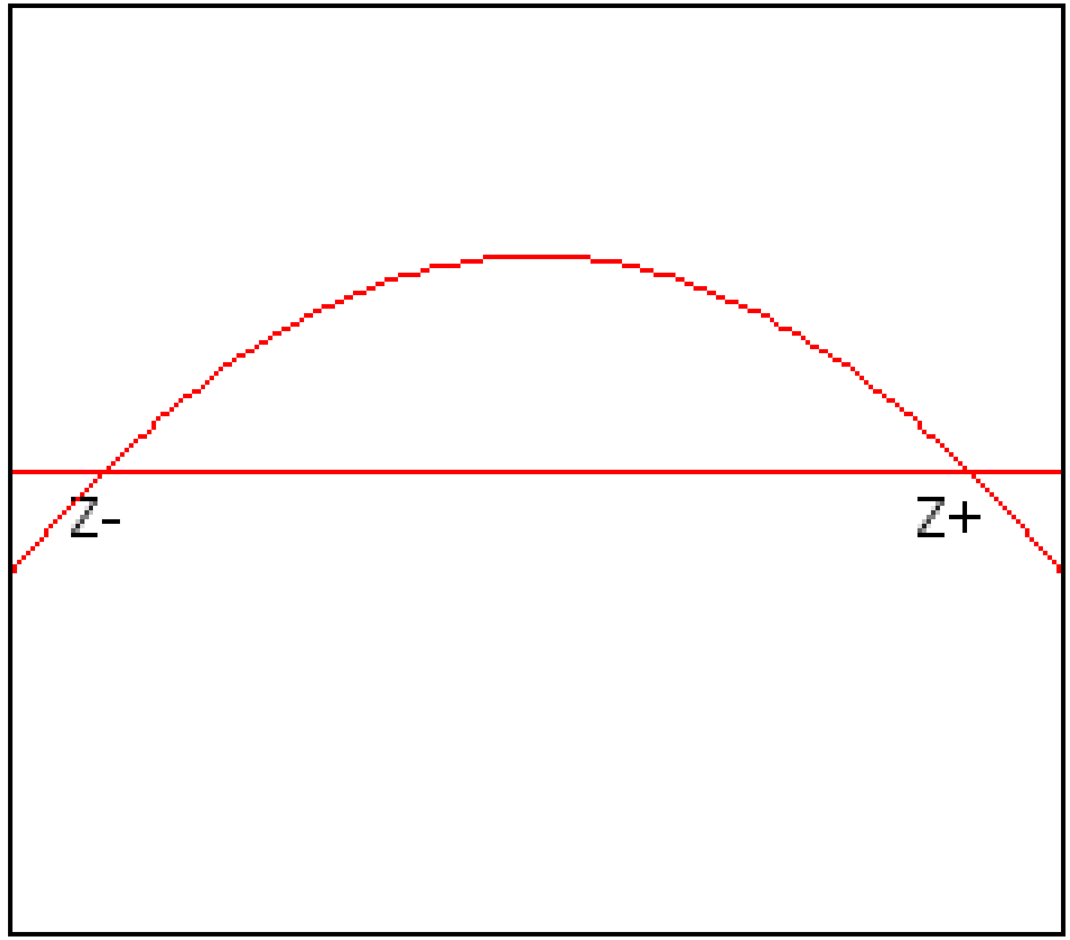
- We are typically interested in the *natural rate of increase* of a population.
- If ζ is constant, then we may drop its notation. We typically assume that θ enters multiplicatively, and we write $G(Z_t, \theta) = Z_t + H(Z_t)\theta$.
- With continuous time, we rewrite as the difference, divide by θ, and take the limit as $\theta \to 0$:

  $$\lim_{\theta \to 0} \frac{Z_{t+\theta} - Z_t}{\theta} = \dot{Z}_t = H(Z_t).$$

  This *defines* H.
- Example: $H(Z) \equiv 0$ is a constant population size. This means renewable resources include exhaustible resources as a special case.
- Example: with abundant resources, $H(Z) = \lambda Z$, giving exponential growth.
- Example: often, $H(Z)$ is a bell curve.

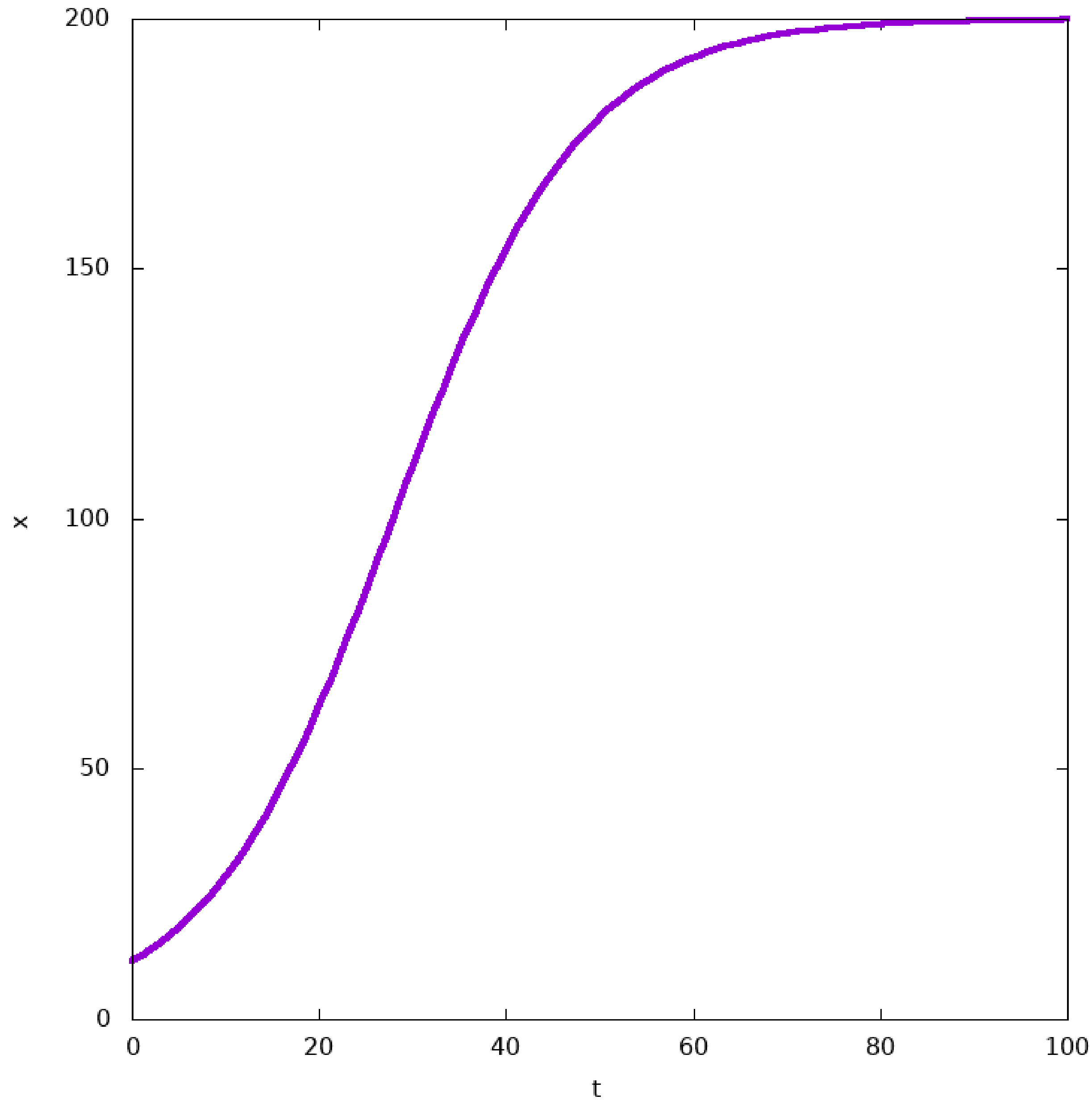
Bell-Shaped Curves

- $H''(Z) < 0$ for all $Z \geq 0$.
- $0 \leq \underline{Z} < \bar{Z}$ such that $H(\underline{Z}) = H(\bar{Z}) = 0$.
- Unique $\hat{Z}$ where $H'(\hat{Z}) = 0$ and $H(\hat{Z}) > 0$.



Logistic Growth

The "S-shaped" logistic growth curve is a typical time path for a bell-shaped natural rate of increase function (in logistic growth, $H(Z) = aZ(1 - bZ)$, an inverted parabola.



Comment on Extreme Cases

- We described pure exhaustible resources as a boundary case of renewable resources.
- Although we can use the same specifications to describe them, the mathematical analysis of the typical case often doesn't hold for the boundary case.
- Example: linear functions are a special case of convex functions. But a linear cost function doesn't allow us to determine the scale of firms in the industry—it doesn't matter if there is one firm or many, the cost structure is the same. However, strictly convex costs (decreasing returns to scale) will have a unique scale for each firm in equilibrium.

Harvesting the Fishery

- How does the dynamic of the fishery change when we add large-scale fishing to the model?
- In economics, we simply remove fish from the population. In a differential equation model, we represent this by the rate of harvest at each instant of time, and write

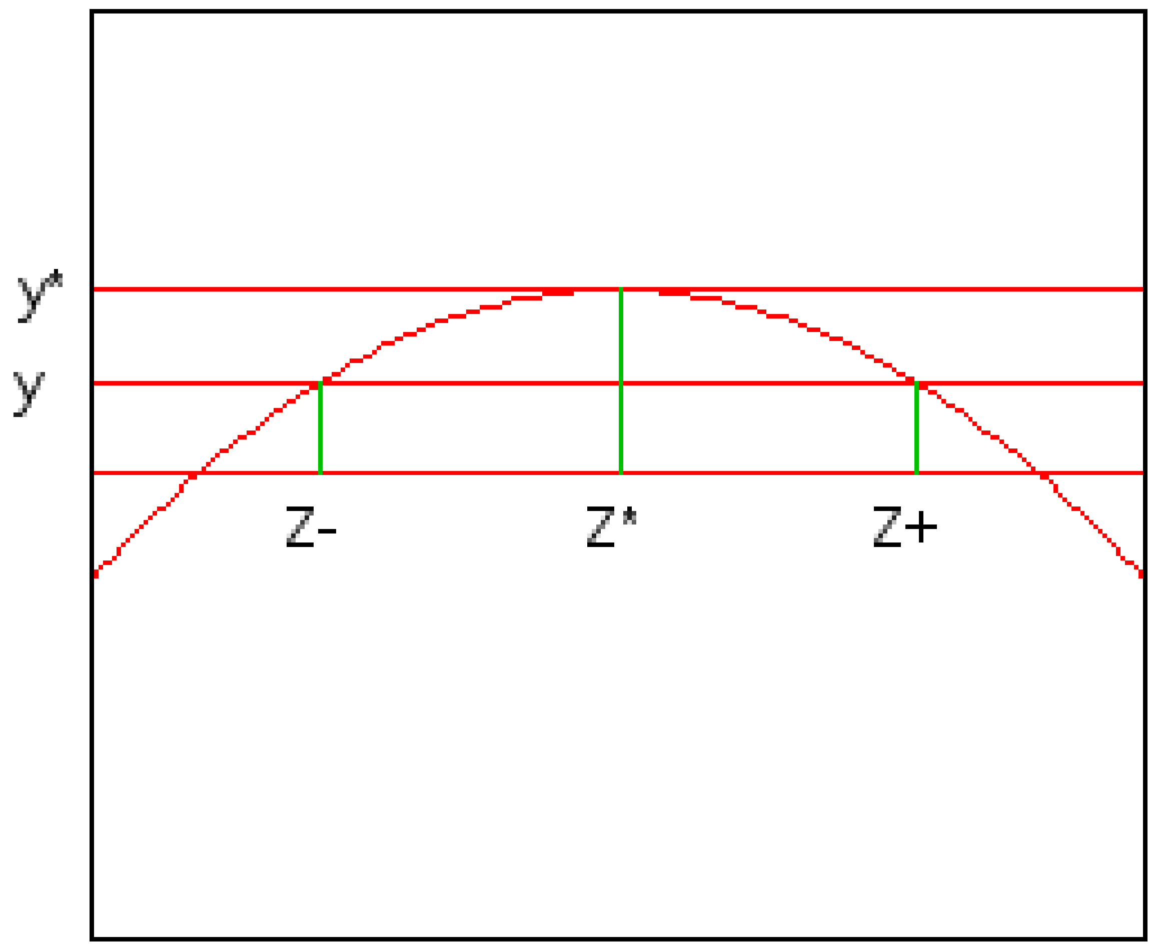
$$\dot{Z}_t = H(Z_t) - y_t.$$

- In biology, we would also consider
  - Collateral damage to predators and prey of the commercially valuable fish. For example, tuna nets often catch (and drown) dolphins.
  - Pollution from fishing activity (*e.g.*, spilled oil and gasoline) might kill the target fish.
  - Fishing activity might interfere with reproduction.

  Our simple model ignores these factors but they could be added.

Constant Harvest Rate

Consider the stationary policy $y_t = y \leq \tilde{y} = \max H$ for all t. Note that there are two steady states at Z^- (unstable) and Z^+ (stable), unless $y = \tilde{y}$, when the unique steady state at $\tilde{Z}$ is *unstable.*



The Case $y = \tilde{y}$

- When $y = \tilde{y}$ there is a steady state at $\tilde{Z}$ which is considered *unstable.*
- Looking closely, we see that although for $Z < \tilde{Z}$ where $\dot{Z} = H(Z) - y < 0$ and Z diverges from $\tilde{Z}$, for $Z > \tilde{Z}$ again $\dot{Z} = H(Z) - y < 0$, and Z *converges to* $\tilde{Z}$.
- Couldn't we say $\tilde{Z}$ is "half-stable" or "stable from the right"? Formally, yes, but as a model it doesn't make sense. We *don't know* why we deviate from $\tilde{Z}$, so it could be up or down. Eventually it will be down, and divergence will occur. If it reaches $\underline{Z}$, extinction occurs (divergence is *permanent*).
- Conceptually (*i.e.*, in modeling) "stable" means "always converges." This approach is *useful* in modeling because we may assume that once the steady state is reached, the system will be "approximately in steady state" *forever.*

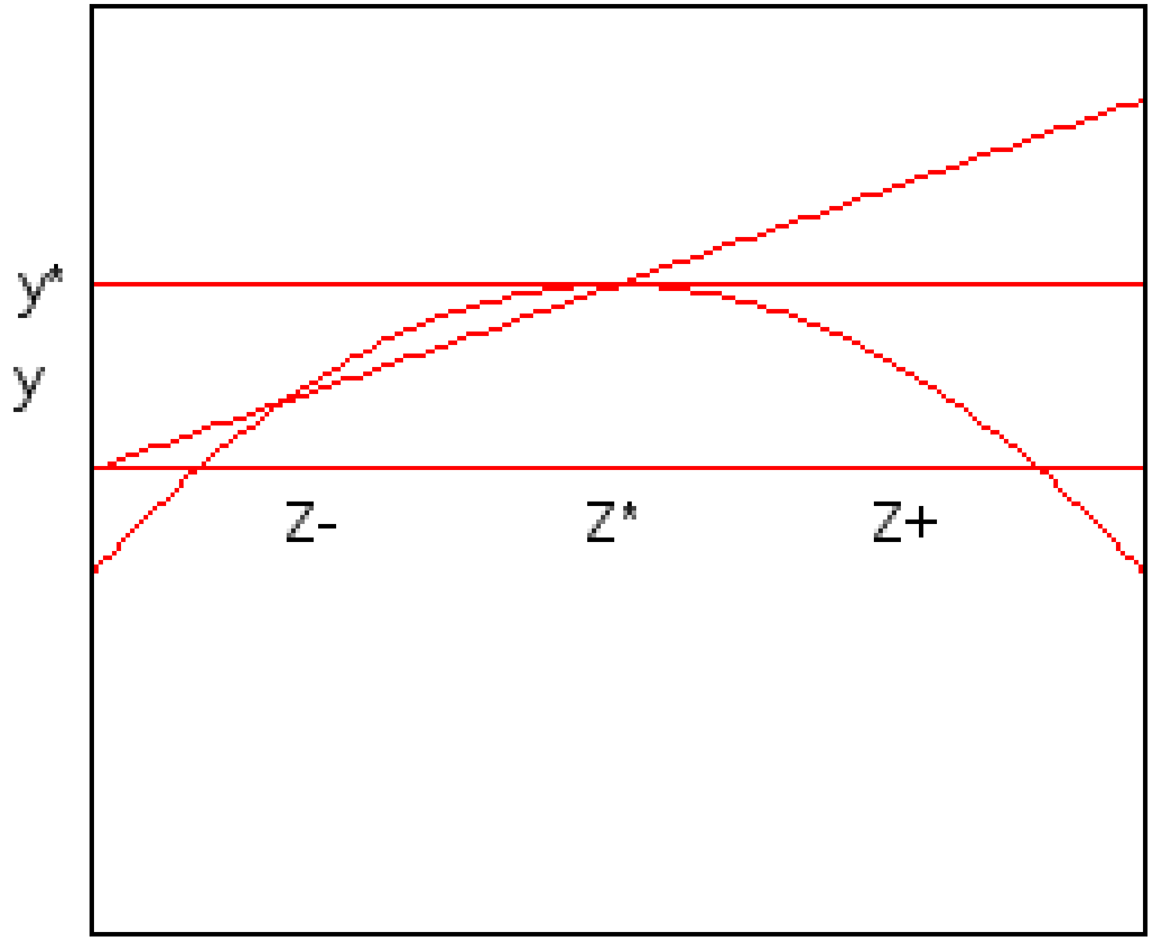
Harvest Policy in a Feedback Loop

- The fact that the maximum harvest steady state is unstable means that a *laissez faire* policy toward the fishery is very *risky*. A policy which ignores the population and simply takes the optimal catch will eventually lead to extinction.
- Use the policy $y_t = \tilde{y} + \xi(Z_t - \tilde{Z})$ to achieve a stable maximum harvest steady state.
  - This policy is called a *negative feedback loop*. The path of Z it induces is a *closed-loop solution*. (It's *negative* because in Z phase space we have

    $$\dot{Z} = H(Z) - h(y) = H(Z) - (\tilde{y} + \xi(Z - \tilde{Z}))$$

    so the direct coefficient on Z is $-\xi$.)
  - The constant policy is called an *open-loop solution* (generally, any policy which is not a function of Z).
  - Note that, though simple to state, this is not necessarily an easy policy to implement: how do you count the tuna in the sea? If many fisherman are involved, how do you share the cost of the census?

Closed-Loop Harvesting



Use the *negative feedback policy* $y_t = \tilde{y} + \xi(Z_t - \tilde{Z})$. Note that the steady state at maximum harvest is *stable*.

Comparing Solow's model with the fishery

- In Solow's model, the characteristic equation results in a bell curve, intersecting the horizontal axis (meaning $\dot{k} = 0$, a *steady state*) at the origin and at some level of the capital-labor ratio, k^*. The steady state at 0 is *unstable*, while the steady state at $k^* > 0$ is *stable*.

- Similarly, in the fishery, the *natural rate of increase* curve is H a bell curve, intersecting the horizontal axis (meaning $\dot{Z} = H(Z) = 0$, a *steady state*) at $\underline{Z} \geq 0$ and at $\bar{Z} > \underline{Z}$. The steady state at $\underline{Z}$ is *unstable*, while the steady state at $\bar{Z}$ is *stable*.

- The difference between the two models is that in the fishery, $H(Z)$ can be converted directly to consumption by harvesting the fish, which means that the maximum of H, $H(\tilde{Z})$ is of economic interest as the maxumim sustainable catch. On the other hand, $\dot{k}$ is not even an input into production (k is), so it is not of economic interest in itself.

When Do Steady-State Policies Make Sense?

- This is an *economic* (not "purely dynamic" as with Solow) issue. Need an objective to optimize (until now, implicitly "maximize steady state y").
- Need an infinite planning horizon, else you want to use up the population "before the world ends."
- Entry must be controlled (else "tragedy of the commons" occurs as new entrants try to "grab their share" by consuming earlier than future entrants do).
- How about "maximum sustainable" harvest $\tilde{y}$ (with a closed-loop policy)?
  - No discounting, or time preference outweighs higher future consumption.
  - With discounting, a steady-state optimal policy exists on the left side of the bell, with $Z < \tilde{Z}$.

A Monopoly Fishery

- Consider a monopolist owning a whole fishery (*e.g.*, in a lake), selling fish at a fixed competitive price, with no variable cost of catching fish.
  - Unreasonable, but in fact a technical assumption: the basic result (maximum sustainable catch is not dynamically optimal) holds for "usual" cost functions.
  - Profit maximization is straightforward, but the same kind of analysis works for social planner too.
- There is a fixed cost: interest on a large loan taken to buy the fishery. Interest is charged on the remaining principal. Also suppose the fishery was at its stable steady state population when purchased.

The Case $Z > \tilde{Z}$

- As long as $Z > \tilde{Z}$, the monopolist will want to catch fish as fast as he can. The argument is straightforward, there is no tradeoff:
- He could take the same fish now or in the future: catching now reduces the loan, and interest costs in the next period. Else he'll have to pay higher interest for one period, and he has to reduce his loan someday.
- Catching now doesn't reduce the possible catch in the long run, because the population is currently growing slower than its maximum rate—catching fish faster than the natural rate of increase $H(Z)$ actually increases the steady state yield.

The Case $Z \leq \tilde{Z}$

- At $\tilde{Z}$, catching a tiny amount more than $H(\tilde{Z})$ decreases H by "almost nothing" (that's what "$H' = 0$" means) while applying the revenue from the catch to decrease in principal owed means that interest to be paid decreases by a small amount—but strictly more than nothing.
  - Thus it is profitable to have a stock $Z < \tilde{Z}$ with steady state yield less than maximum because the fixed cost of the interest payment decreases more.
  - Define Z^* to be the optimal steady state stock accounting for interest payments.
- Computing Z^* case requires a precise notion of "marginal productivity" to compare to the marginal cost of interest. We approach this via the concept of arbitrage and the return to waiting defined as the "own rate of return".

Arbitrage

- The most important concept in finance is *arbitrage*. That is, exploiting a configuration of markets that allows you to buy a good at a low price in one market and immediately sell it in another, guaranteeing a profit.
- Buying low, selling high among separated markets for real commodities involves transportation and storage, *i.e.*, production. These aren't *pure* arbitrage; *pure arbitrage* really applies only to financial markets.
- There can be differences between prices in geographically separated markets (such as for the dollar or for gold) that are open at overlapping times (*e.g.*, London and New York). They are very quickly erased by arbitrage.
- Arbitrage is very useful for understanding pricing of real assets by comparison to specific interest rates. We sell some of the asset whose value grows more slowly and buy some of the asset whose value grows more quickly.
- We will determine Z^* by arbitrage of $H'(Z)$ against the interest rate r.

Details of the arbitrage

- Consider the units of r and H'. r is measured in dollars (or yen) of interest paid per dollar (yen) of principal per unit of time, so its unit is 1/(time unit). $H'(Z)$ is measured in fish per fish per unit of time, so its unit is also 1/(time unit). So the units are compatible.

- But what does it mean to sell one to buy the other? Buying your own debt is just paying dollars to the bank. So we need to translate fish to value, and we do that by multiplying by the price p.

- So the rate of change of the value of the population of fish is just $V(Z) = pH(Z)$, and since p is constant, $V'(Z) = pH'(Z)$, which is the cost of selling one extra fish in terms of the value of the stock of fish.

- But that fish is sold at the same price p, so the change in interest to be paid is rp.

- So there's no profit to be made from the trade of fish for principal or vice versa exactly when $rp = pH'(Z)$, or $r = H'(Z)$.

Rates of Return, Discounting, and Interest

- A central concept in finance and dynamic economics is the return to waiting, usually measured by an interest rate.
- An *interest rate* (strictly defined) is a rate of compensation for use of a sum of money for a given period of time.
- (Market) interest rates equilibrate the time preferences of consumers and the productivity of firms.
- A consumer's time preference is expressed in terms of a *discount rate*, which has the same units as an interest rate.
- Productivity of investment by firms is expressed in terms of the *rate of return*, which also is formally the same as an interest rate.

Own Rate of Return

- We can naturally measure the value of saving/investing, *i.e.*, not harvesting the population, even if there is no market price or interest rate.
- Measure the value in terms of the good itself (rather than money).
- Suppose the good is "self-reproducing": given a stock of the good Z, after an interval of θ there will be $Z + G(Z, \theta)$, for some function G. (This is the same G used to define H.)! Examples: $G_Z(Z, \theta) < 0$ for all Z – an "iceberg," or $G_Z(Z, \theta) > 0$ – a bank account, or any shape – living population.
- Suppose we hold one more unit of the good. Then the extra good in the following period, over and above the unit we hold, is approximately $G_Z(Z, \theta)$. (*I.e.*, the total amount extra in the next period is $1 + G_Z(Z, \theta)$.)
- Not $G(Z + 1, \theta)$, which is the return to the entire population Z. We're interested in the amount of increase due to the extra investment: $\frac{G(Z+\delta,\theta)-G(Z,\theta)}{\delta}$, which when taken to the limit as $\delta \to 0$ is $G_Z(Z, \theta)$.

A Caution

- Regarding own rate of return, it is sometimes claimed that the population would change, and that causes difference between G_Z and G. Not true: the point here is the usual difference between *average* and *marginal* which is central in modern economics.
- We want to compare an increment of δ to investment, then we have $G(Z + \delta, \theta)$ next period, the net gain is $G(Z + \delta, \theta) - G(Z, \theta)$.
- Dividing by δ and taking δ to zero gives the definition of the derivative of G, *i.e.*, G_Z.

The $Z^* < Z \leq \tilde{Z}$ Case

- At $Z = \tilde{Z}$, the argument for $Z > \tilde{Z}$ applies "at the margin": the interest rate is strictly greater than zero, the principal has to be paid sometime, and the maximum steady state isn't quite decreasing.
- For $Z < \tilde{Z}$, if Z is close enough to $\tilde{Z}$, the maximum steady state catch decreases very little, and it can be made smaller than the interest payment at the specified rate (a constant).
  - Thus it makes sense to decrease steady state catch in return for decreasing principal, and so reducing interest paid.
- When is Z "small enough" for interest to be larger than the lost steady state harvest? That is determined by the own rate of return $H'(Z^*)$: by the usual "marginal argument" the benefit of reduced interest is just offset by the cost of lost steady state harvest when $H'(Z^*) = r$ (the interest rate).

The Whaling Controversy

- The blue whales were hunted to near extinction (in the conservationist sense) by the early 1960s.
- We have a lot of data (appended) from the Committee for Whaling Statistics of Norway.
- Looking at the data we see that efficiency of catch was greatest in the 1930s, by the late 1950s was much less efficient.
- We can also see that the numbers are hardly smoothly varying, it's hard to see a clear trend, other than the average productivity.

The Whaling Controversy, cont.

- So we see that the Japanese, Icelanders, and Norwegians argued that $Z_0 \geq \tilde{Z} > \underline{Z}$.
- The conservationists (and the non-whaling nations) worried that $Z_0 < \underline{Z}$, that is $Y = F(Z, X^*(Z)) > H(Z)$ for all Z
  - $F(Z, X)$ is the catch of whales as a function of number of whales and number of ships in the industry (production function), and $X^*(Z)$ is the zero-profit number of ships as a function of Z (competitive equilibrium).
- Theoretically, it's impossible to say which is correct; we look at an empirical model.

An Empirical Model

- Using a discrete model, we have
  - $Z_{t+1} - Z_t = H(Z_t) = Z_t^{\alpha}(A - Z_t^{1-\alpha})$, and
  - $Y_t = F(Z_t, X_t) = AZ_t^{\alpha}(1 - e^{-\nu X_t})$.
- Based on the data, we can estimate this system of equations (done by Michael Spence). We get

$$\nu = 0.0019, \quad \alpha = 0.8204, \quad A = 8.356.$$

- Maximum sustainable catch $H(\hat{Z}) = 9890$, while $\hat{Z} = 45177$.

Are the Whales Endangered?

- Using $Y_{1960} = 1987$ and $X_{1960} = 418$, we get an estimate of $Z_{1960} = 1639$. That is, at that pace the whales would be extinct in the end of 1960! (Estimating for 1955-1959 gives $Y = 1636$, 1496, 1651, 1105, and 1174 respectively.)
- These numbers are pretty suspicious: they claim that for five years running whalers took more whales than were in the ocean.
- The implied natural rates of increase are pretty impressive, though: $H(Z) = 1983$, 1867, 1995, 1518, 1582, and 1985. Note that these track the catch numbers quite closely. This is not surprising: the model is tuned that way.

Optimal Policy

- Our model implies that the whales are not below $\underline{Z}$, but well below Z^*.
- In fact, the calculation of the optimal policy suggests (using the figures above and $r = 0.05$) a period of abstaining for 9 years.
- $Z^* = 67000$, and $H(Z^*) = 9000$.
- History showed that political aspects are extremely important.

Whaling Data (pre-WWII)

Year	Boats	Catch	Year	Boats	Catch	Year	Boats	Catch
1909	149	316	1920	112	2987	1931	100	6705
1910	178	704	1921	142	5275	1932	186	19067
1911	251	1739	1922	174	6869	1933	199	17486
1912	246	2417	1923	194	4845	1934	242	16384
1913	254	2968	1924	234	7548	1935	312	18108
1914	182	4527	1925	235	7229	1936	254	14636
1915	151	5302	1926	233	8722	1937	357	15035
1916	94	4351	1927	222	9676	1938	362	14152
1917	130	2502	1928	242	13792			
1918	141	1993	1929	337	18755			
1919	154	2274	1930	280	26649			

Whaling Data (post-WWII)

Year	Boats	Catch	Year	Boats	Catch
1945	158	3675	1953	368	3009
1946	246	9302	1954	386	2495
1947	307	7157	1955	419	1987
1948	348	7781	1956	395	1775
1949	382	6313	1957	417	1995
1950	468	7278	1958	420	1442
1951	430	5436	1959	399	1465
1952	379	4218	1960	418	1987

Conclusions

- Profit maximization is consistent with some degree of conservation; it does not imply extinction of the species. Population will be smaller than $\bar{Z}$, but greater than 0 in the steady state.
- If population is depleted below the optimal stationary policy, a profit-maximizing industry would advocate a total ban on harvesting, just like the conservationists!
- The maximum sustainable rate is not a focal point for policy.
- If $\tilde{Z} < \underline{Z}$, profit maximization conflicts with conservation.
- Also, things may be somewhat harsher than it seems here: in the long run, as supply of output falls, you might expect that q would rise, causing harvest rates to rise.
- $\tilde{Z} < \hat{Z}$.

Homework 4: due 2020-10-30, 17:00

1. Explain why the "amount extra" when we discuss the *own rate of return* is $1 + G_Z(Z, \theta)$, not $\theta + G_Z(Z, \theta)$. (You don't need to know calculus to explain this.)

2. We argued that the optimum (profit-maximizing) steady state occurs at a catch $\tilde{y} = H(\tilde{Z})$ such that with interest rate r, $H'(\tilde{Z}) = r$. Give the corresponding expression in terms of G. (This requires knowledge of calculus to understand, but you may be able to guess even if you don't know much calculus from the expression defining the derivative of G.)

3. Compare the right-hand side of Solow's characteristic equation $\dot{k} = sf(k) - (n + d)k$ to the H curve in the fishery model.

   (a) Check that the right-hand side of Solow's characteristic equation is a *bell curve*.

   (b) What points in the graph of Solow's characteristic equation are the analogues of $\underline{Z}$ and $\bar{Z}$ in the fishery model?

Homework 5: due 2020-10-30, 17:00

Recall Michael Spence's model of the blue whale fishery. It is a discrete model, where Z denotes population and X the number of whaling ships, the natural rate of increase is defined

$$Z_{t+1} - Z_t = H(Z_t) = Z_t^{\alpha}(A - Z_t^{1-\alpha}),$$

and the industry catch function is

$$Y_t = F(Z_t, X_t) = AZ_t^{\alpha}(1 - e^{-\nu X_t}).$$

Spence estimated the parameters of this system to be

$$\nu = 0.0019, \quad \alpha = 0.8204, \quad A = 8.356.$$

1. Spence calculated that the maximum sustainable catch is $H(\tilde{Z}) = 9890$, where $\tilde{Z} = 45177$. Verify these calculations.
2. What is the stable steady state population of whales $\bar{Z}$ in this model, assuming catch is $Y = 0$?
3. What is the "point of no return" $\underline{Z}$, assuming catch is $Y = 0$?
4. What is the "point of no return" $\underline{Z}$, assuming catch is $Y = 1500$?