Economic Dynamics

Stephen Turnbull

Department of Policy and Planning Sciences

Lecture 10: December 14, 2018

Abstract

More on the overlapping generations models.

Overlapping generations models

- In an *overlapping generations* (OLG) *model*, there are constraints between agents existing at the same time and a given agent across time periods.

A simple OLG model

Follows Ch. 17 of Lucas and Stokey.

- The economy has a constant population of worker/consumers. We use a model with a single representative (population of 1 per generation), which doesn't affect the results because of assumptions of linearity in the objectives and constraints, and competitive markets.
- Each agent lives for two periods, working when young and consuming when old.
- There is a single, non-storable good, produced with a linear technology $y = xl$, where X is generated by a *Markov process*.
- The utility function is $U(c, l) = -H(l) + V(c)$.
- There is a constant supply of *fiat money* M, the only storable good in the model.

How the OLG model works

- Based on an assumption of equilibrium, markets will clear:
  - The young worker supplies labor l, produces $y = xl$, and receives all the money M from the old consumer at the market price.
  - The old consumer will consume $c = y$, and pay all the money M to the young worker.
- The consumer good equilibrium is the consumers compete to buy the good, so bid up price until all the money is spent, while the workers compete to sell all the good: $py = M$.

The worker's model

- The components of utility H and V satisfy
  - $H'(l) > 0$ and $H''(l) > 0$ for all l,
  - $H'(0) = 0$ and $\lim_{l \to L} H'(l) = \infty$ (Inada!),
  - $V'(c) > 0$ and $V''(c) < 0$ for all c.
- The equilibrium is characterized by
  - the price $p(x)$, which depends on the state of the world (random worker productivity),
  - the "labor supply" function $n(x)$ (n depends on x, not the wage), and
  - goods-market-clearing $xn(x) = M/p(x)$.
  - The labor market also clears by *Walras' Law*.
- When old, the worker born at t consumes $x_t n(x_t)(p(x_t)/p(x_{t+1}))$.

The worker's optimization

- The worker chooses $n(x)$ to maximize

$$-H(l) + \mathcal{E}_\xi \left[V(xl\frac{p(x)}{p(\xi)}) \mid x \right]$$

  where the worker knows her own productivity x but the productivity of the next generations is random ξ.

- Given a price function p, the first-order condition for n is given by solving

$$H'(n(x)) = \mathcal{E}_\xi \left[x\frac{p(x)}{p(\xi)} V'(xn(x)\frac{p(x)}{p(\xi)}) \mid x \right]$$

  (there are no n' because x is a parameter known to the worker, not a choice variable).

- Substituting from the market-clearing conditions for this period and next gives

$$n(x)H'(n(x)) = \mathcal{E}_\xi \left[\xi n(\xi) V'(\xi n(\xi)) \mid x \right]$$

The equilibrium

- Suppose x has a distribution independent of time and across time. Then $n(x) = \bar{n} > 0$ for all x.

- Under certain conditions on the Markov process, and the same assumptions on production and utility, for a general process (*i.e.*, serially correlated x), there exists

$$f^*(x) = \mathcal{E}_\xi \left[\phi(\xi \zeta^{-1}(f^*(\xi))) \mid x \right]$$

  where $\phi(y) = yV'(y)$ and $\zeta(l) = lH'(l)$. **Note:** f^* is defined as a fixed point, like a value function.

The Lucas "Islands" model

We change the preceding model in the following way.

- We have a *deterministic* production function, $y = l$ (*i.e.*, $x \equiv 1$).
- There are two "islands" between which the population is split. Workers (young) are assigned to the two islands such that $\frac{\theta}{2}$ go to one island and $1 - \frac{\theta}{2}$ to the other, where θ is random between $0 < \underline{\theta} < \bar{\theta} < 2$.
- Consumers (old) are assigned randomly to the two islands such that each island has half the old population and half the money.
- The government pays interest on or taxes the money stock randomly, such that $m_{t+1} = xm_t$ for a worker who received m_t, and x is random between $0 < \underline{x} < \bar{x} < \infty$.
- The varying ratio of workers to consumers is a *real* shock (affects available consumption per person in the old generation), while the monetary shock is *nominal*. *Nominal* means it doesn't change the physical possibilities, only the ability of workers to make an accurate assessment of future consumption, and thus their incentive to work.

Market conditions

- Here the *state* of the economy is 2-dimensional: (x, θ). (x is the increase factor for the money supply, θ the population assignment between islands.)
- As before (equilibrium) price of consumption and (optimal) labor "supply" are functions of the state: $p(x, \theta)$ and $n(x, \theta)$.
- In the previous model, $pc = \frac{M}{l}$, where the latter is constant, so we can invert the equilibrium price function $x = p^{-1}(p)$, and it doesn't matter if the workers can observe x, by assumption of competition they know p and can deduce x from that.

  Here, M is *uncertain*, so in equilibrium, by observing p and given x you can figure out θ, and *vice versa*. But you can't deduce both.
- Assume x and θ independent for each t, and (x, θ) *i.i.d.* over time.

Equilibrium conditions

- Labor supply $n(x,\theta)$ maximizes over l

$$-H(l) + \mathcal{E}_{\bar{x},\bar{\theta}} \left\{ \mathcal{E}_{x',\theta'} \left\{ V \left[\frac{x' l p(\bar{x},\bar{\theta})}{\bar{x} p(x',\theta')} \right] \mid p(\bar{x},\bar{\theta}) = p(x,\theta) \right\} \right\}$$

- Market clearing: for all (x,θ)

$$n(x,\theta)p(x,\theta) = \frac{x}{\theta}$$