Economic Dynamics

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Abstract

More on the overlapping generations models.

Overlapping generations models

- Up to the present, we've considered dynamic constraints on single homogeneous entities. Examples:
  - In Solow's model, the single "interesting" entity is the *representative worker/consumer*, which we derive using the special properties of CRTS production.
  - In the fishery, the "interesting" entity is the *population* of fish (or whales). Although the fisherman do interact in equilibrium, the dynamic constraint is on the population.
- By contrast, in an *overlapping generations* (OLG) *model*, there are constraints between agents existing at the same time and a given agent across time periods.

A simple OLG model

Follows Ch. 17 of Lucas and Stokey.

- The economy has a constant population of agents (worker/consumers).
- The agent lives for two periods, working when young and consuming when old. (This is a *technical* assumption, convenient in notation, computation, and interpretation because the number of workers equals the number of consumers equals half the population.)
- The utility function is $U(c, l) = -H(l) + V(c)$.
- There is a single, non-storable good, produced with a linear technology $y = xl$, where X is generated by a *Markov process*. (This means that x_{t+1} is generated by a random variable which may depend on x_t but nothing else.)
- There is a constant supply of *fiat money* (government-issued, as with yen and dollars) M.

How the OLG model works

- We make the *technical* assumption that there's one person in each generation. (Like Solow's model, this one is CRTS.)
- Based on an assumption of equilibrium, markets will clear:
  - The young worker will supply labor l, produce $y = xl$, and receive all the money M from the old consumer.
  - The old consumer will consume $c = y$, and pay all the money M to the young worker.
- The old consumer's behavior is forced: they have money, they buy the good in a competitive market, so they'll spend all the money and buy all the good.

The worker's model

- When young, the worker dislikes working, with the usual "decreasing returns to scale" conditions: $H : [0, L) \to R_+$ satisfies
  - $H'(l) > 0$ and $H''(l) > 0$ for all l, and
  - $H'(0) = 0$ and $\lim_{l \to L} H'(l) = \infty$ (Inada!).
- When old, the consumer likes consuming, with decreasing marginal utility. $V : R_+ \to R_+$ satisfies
  - $V'(c) > 0$ and $V''(c) < 0$ for all c.
- The equilibrium is characterized by
  - the "price" (of money in goods, not the reverse!) $p(x)$, which depends on the state of the world (random worker productivity),
  - the "labor supply" function $n(x)$ (n depends on x, not the wage), and
  - market-clearing $xn(x) = M/p(x)$.
- When old, the worker born at t consumes $x_t n(x_t)(p(x_t)/p(x_{t+1}))$.

The worker's optimization

- The worker chooses $l = n(x)$ to maximize
$$-H(l) + \mathcal{E}_\xi \left[V(xl\frac{p(x)}{p(\xi)}) \mid x \right]$$
where the worker knows her own productivity x (by inverting the price function p) but the productivity of the next generations is random ξ.
- This is not a differential equation model. $p(x), n(x)$ are determined "independently" (in a sense) from $p(x'), n(x')$ for $x \neq x'$.
- Given a price function p, the first-order condition for n is given by solving
$$H'(n(x)) = \mathcal{E}_\xi \left[V'(xn(x)\frac{p(x)}{p(\xi)}) \mid x \right]$$
(there are no n' because x is a parameter known to the worker, not a choice variable—the worker chooses a different n for each x).
- Substituting from the market-clearing conditions for this period and next gives
$$n(x)H'(n(x)) = \mathcal{E}_\xi \left[\xi n(\xi) V'(\xi n(\xi)) \mid x \right]$$

- Suppose x has a distribution independent of time and across time. Then $n(x) = \bar{n} > 0$ for all x.

The equilibrium

- Suppose x has a distribution independent of time and across time. Then $n(x) = \bar{n} > 0$ for all x.

- Under certain conditions on the Markov process, and the same assumptions on production and utility, for a general process (*i.e.*, serially correlated x), there exists

$$f^*(x) = \mathcal{E}_\xi \left[\phi(\xi \zeta^{-1}(f^*(\xi))) \mid x \right]$$

  where $\phi(y) = yV'(y)$ and $\zeta(l) = lH'(l)$. **Note:** f^* is defined as a fixed point, like a value function.

The Lucas "Islands" model

We change the preceding model in the following way.

- We have a *deterministic* production function, $y = l$ (*i.e.*, $x \equiv 1$).
- There are two "islands" between which the population is split. Workers (young) are assigned to the two islands such that $\frac{\theta}{2}$ go to one island and $1 - \frac{\theta}{2}$ to the other, where θ is random between $0 < \underline{\theta} < \bar{\theta} < 2$.
- Consumers (old) are assigned randomly to the two islands such that each island has half the old population and half the money.
- The government pays interest on or taxes the money stock randomly, such that $m_{t+1} = xm_t$ for a worker who received m_t, and x is random between $0 < \underline{x} < \bar{x} < \infty$.
- The varying ratio of workers to consumers is a *real* shock (affects available consumption per person in the old generation), while the monetary shock is *nominal*. *Nominal* means it doesn't change the physical possibilities, only the ability of workers to make an accurate assessment of future consumption, and thus their incentive to work.

Market conditions

- Here the *state* of the economy is 2-dimensional: (x, θ). (x is the increase factor for the money supply, θ the population assignment between islands.)
- As before (equilibrium) price of consumption and (optimal) labor "supply" are functions of the state: $p(x, \theta)$ and $n(x, \theta)$.
- In the previous model, $pc = \frac{M}{l}$, where the latter is constant, so we can invert the equilibrium price function $x = p^{-1}(p)$, and it doesn't matter if the workers can observe x, by assumption of competition they know p and can deduce x from that. p Here, M is *uncertain*, so in equilibrium, by observing p and given x you can figure out θ, and *vice versa.* But you can't deduce both.
- Assume x and θ independent for each t, and (x, θ) *i.i.d.* over time.

Equilibrium conditions

- Labor supply $l = n(x,\theta)$ maximizes over l

$$-H(l) + \mathcal{E}_{\bar{x},\bar{\theta}} \left\{ \mathcal{E}_{x',\theta'} \left\{ V \left[\frac{x' l p(\bar{x},\bar{\theta})}{\bar{x} p(x',\theta')} \right] \mid p(\bar{x},\bar{\theta}) = p(x,\theta) \right\} \right\}$$

  where

  - x, θ are the current values, but the worker does not know them
  - The worker does know p, so can deduce that all $\bar{x}, \bar{\theta}$ such that $p(\bar{x},\bar{\theta}) = p(x,\theta)$, and so take expectation over only those values of $\bar{x}, \bar{\theta}$, and
  - given $\bar{x}, \bar{\theta}$, the worker can take the expectation of the consumption next period based on the independent distribution of x', θ', the values of the nominal and real shocks respectively.

- Market clearing: for all (x, θ)

$$n(x,\theta)p(x,\theta) = \frac{x}{\theta}$$

Interpretation

- The utility function looks very complicated because of all the bars and primes, but the important aspects are
  - the description of each on the previous slide, and
  - most important, the conditioning equation $p(\bar{x}, \bar{\theta}) = p(x, \theta)$, which shows how the nominal shock and the real shock are confounded (confused) by a rational consumer/worker.
- The end result is that we can show that $\frac{dn}{dp} > 0$, which is a Philips curve, *i.e.*, a positive relationship between employment and inflation.