Economic Dynamics / 経済動学

Midterm Examination / 中間試験

November 10, 2017 / 平成２９年１１月１０日

General instructions / 一般説明

Japanese translations may follow the English text. If the English and Japanese versions of any text differ in meaning, the English text is correct. Please ask for clarification if you have any doubt. Any corrections will be posted.

英文に次いで和訳がある場合がある。英文と和文の間に食い違いがあれば、英語の方が正しい。ただし、不明な点については遠慮なく聞いてください。あらゆる訂正は掲示します。

Be sure to write your name and student ID number on each sheet.

名前と学籍番号を忘れずに各ページに記入してください。

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** In Japanese, please take great care in writing kanji. Avoid abbreviated kanji; the only one I know is the 3-stroke mongamae.

以下に経済動学の知識を検定する問題のすべてに解答せよ。**解答の言語は日本語でも英語でも構わない。**もし日本語で書けば漢字などの書き方に十分注意してください。たとえ、省略した漢字などを使わないこと。（私が読めない場合には省略した文字を「間違え」と採点する。）

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited. Some dictionaries may be provided at the examination.

ノート・教科書・辞書・電卓・携帯電話・その他のメモリを持つ電子製品の用は禁止である。全ての計算は簡単であるので電卓などは必要ない。辞書を数冊用意する。

As usual, the only items that should be present on your desk are pencils, pens, erasers, pencil sharpener, watch, and the examination paper. Put other items in your bag and place the bag under the seat or desk, or on the seat next to you.

机の上にペン・鉛筆・消しゴム・鉛筆削り・時計・この試験用紙の他の物を置かないこと。その他のものを側の席に置くこと。

Instructions are continued on the back of this page.

ページの裏に説明が続く。

General instructions, cont. / 一般説明の続き

Except for calculations, I can give complete answers to most problems within 3 lines. Some questions can be answered correctly with 2 or 3 words. Students usually more space to express their answers, but you should try to avoid making these problems harder than I intend them to be. Answers will be evaluated on presence of the correct idea, not the quantity of words used.

計算問題以外の問題には私は３行以内十分に答えられる。少数の問題は６文字でも可能だ。もちろん学生の方がより多くのスペースを要するが、私の意味より難し過ぎることを考えないでください。採点ははっきり正しい概念が含まれているかどうかによって判断し、それ以上に文字の数は構わない。

Below each problem enough space is provided for a complete answer. Please write your answers there. If you need more space for an answer, use any available empty space. Clearly indicate where the additional text is, and label it clearly with the question it answers. A figure with axes is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used, and if needed, how you derived them.

各問題の下に十分にスペースを用意するのでそこに買いてください。それ以上のスペースが欲しかったらどの空白でも構わない。ただし、用意したスペースに「追加あり」を示し、追加のテキストにどの問題の答えかをはっきり表すこと。グラフ問題には軸を用意するのでそれを使用すること。計算問題には結果だけは少数点（０可）しか与えなく、使用した式などが必要だ。場合により、導き出す方法も表すこと。

Problems - 問題

Each question is worth 10 points, unless otherwise specified.
原則として各問は１０点満点だ。例外の場合だけに点数を提示する。

1. `[Problem ID #29] Solow model; neutral progress`
   Recall that in the Solow model with (Harrod-neutral) technological progress, the characteristic equation is $\dot{k} = sf(k) - (n + d + \lambda)k$. Name and briefly describe each of the following symbols from the characteristic equation:
   （ハロッド中立）技術進歩を含むソーロモデルでは基本方程式は $\dot{k} = sf(k) - (n + d + \lambda)k$ だ。以下の記号の名前に短い説明を書け。

   (a) k
   k *is* K/L *the capital-labor ratio (or "per-capita capital"). It is the* state variable *of Solow's model.*

   (b) s
   s *is the saving rate in the economy, or the share of output devoted to investment in capital.*

   (c) n
   n *is the population growth rate,* $\dot{L}/L$*. It is a pure number since it is measured relative to the population.*

   (d) λ
   λ *is the rate of labor-enhancing technological progress, or the growth rate of labor productivity.*

   (e) What is the **steady state growth rate** of per capita consumpion $c = \frac{C}{L}$?
   一人当りの消費 $c = \frac{C}{L}$ の**定常状態での成長率**を書け。
   It is n*, because* all *macro variables grow at that rate in steady state (that is why it is called "balanced growth.")*

2. `[Problem ID #107] CRTS computation`
   A neoclassical, constant returns to scale (CRTS) production function is a crucial assumption in growth theory.
   ネクラシカルCRTS生産関数は経済成長論に不可欠な前提です。

   (a) For a generic production function $F(K, L)$, write the condition it must satisfy to be considered CRTS.
   一般的生産関数$F(K, L)$がCRTSになる条件を書け。

   $\lambda F(K, L) = F(\lambda K, \lambda L)$.

   (b) For the Cobb-Douglas production function, $F(K, L) = AK^{\alpha}L^{1-\alpha}$, derive the *per-capita production function* $f(k)$.

カブダグラス生産関数$F(K,L) = AK^{\alpha}L^{1-\alpha}$の一人当 りの生産関数$f(k)$を導き出せ。

$f(k) = F(k,1) = Ak^{\alpha}1^{1-\alpha} = Ak^{\alpha}$.

3. `[Problem ID #42] growth: Solow; convergence: draw OECD freehand`

Consider the *convergence hypothesis* for the OECD, which *does* show historical evidence for the convergence hypothesis.

(a) Give the convergence hypothesis.
収束仮説を書け。

The convergence hypothesis states that the parameters of the Solow model should be quite similar for different countries, and since there is a unique stable steady state, in the long run all economies should converge to that steady state, differing only in population and the other macro variables (which are all proportional to population).

(b) Explain why the convergence hypothesis seems likely to be true, referring to the parameters of the characteristic differential equation.
なぜ収束仮説があいろうかを説明せよ。成長の微分方程式のパラメーターに触れること。

Technology is knowledge and skill, and all human beings are capable of acquiring that knowledge and skill, so the production function f and the depreciation rate d should be common to all countries. While savings rates and population growth rates vary substantially, the "advanced" countries seem to have similar parameters, so we may expect the emerging market economies to converge there as well.

(c) Draw "before" and "after" graphs of growth rate ($\dot{y}/y$) and income (y, where the unit is US real per-capita income in each period) for four countries whose situation is similar to the OECD.
OECDの状況を似ている 4 ヶ国の成長率 ($\dot{y}/y$) 対所得 (y、ただし単位が米一人当り実質所得) を書け。

4. `[Problem ID #94] simple fishery; phase diagram analysis; logistic`

(30 points) We discussed the *fishery* model of a single population as renewable resource with constraints on growth. Consider the *logistic model* of population growth:

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z).$$

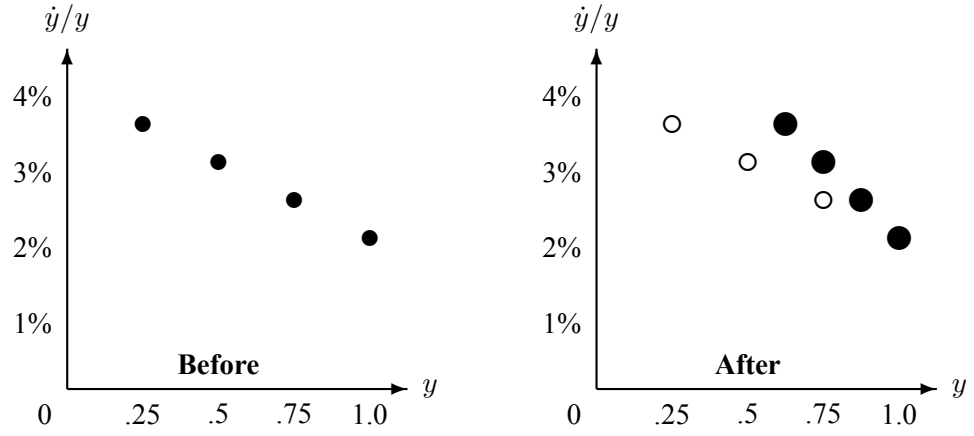


Figure 1: Convergence examples

漁業での魚母団の成長制限された再生可能資源としたモデルを講義で触 れた。母団成長のロジスティックモデルを検討しよう：

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z)$$

(a) What do the expressions a and bZ represent? (Hint: it may be useful to expand the expression given for $H(Z)$.)
a と bZ という式は何を表現するか。（ヒント： $H(Z)$ の別の式が役立つかも。）

a is the birth rate as a fraction of the population. bZ is the death rate. Note that bZ depends on the population and is increasing, so there is a (negative) congestion externality to population.

(b) What can you say about the case $b = 0$?
$b = 0$ の場合には何が言えるか。

This is the unconstrained population growth case. You can interpret a as the difference between the birth rate and the death rate.

(c) Show that H satisfies the four conditions for a *bell curve*.
H がベル曲線の 4 条件を満たすことを証明せよ。

$0 = \underline{Z} < \bar{Z} = \frac{a}{b}$, where $H(\underline{Z}) = H(\bar{Z}) = 0$. $Z^* = \frac{a}{2b}$ satisfies $H'(Z^*) = 0$ and $\underline{Z} < Z^* < \bar{Z}$.
$H'(0) = a > 1$, and $H''(Z) = -2b < 0$.

(d) Solve for the populations $\underline{Z}$ and $\bar{Z}$ such that $H(\underline{Z}) = H(\bar{Z}) = 0$.
母団数 $\underline{Z}$ と $\bar{Z}$ は $H(\underline{Z}) = H(\bar{Z}) = 0$ を満たす。 $\underline{Z}$ と $\bar{Z}$ の解を求めよ。

Separating the factors, we have $Z = 0$ *and* $1 - \frac{b}{a}Z = 0$. *So* $\underline{Z} = 0$ *and* $\bar{Z} = \frac{a}{b}$.

(e) What is special about the populations $\underline{Z}$ and $\bar{Z}$?
母団数 $\underline{Z}$ と $\bar{Z}$ の特徴を説明せよ。

They are steady states.

(f) Solve for the population Z^* such that $H'(Z^*) = 0$.
$H'(Z^*) = 0$ を満たす母団数 Z^* の解を求めよ。

Done in part **??**.

Now consider the basic model of a constant level of extraction $y \geq 0$:

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z) - y.$$

(This is also called an "open loop" dynamic system.)
収穫を定数 $y \geq 0$ で常に取る基本的モデルを考えよう：

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z) - y$$

（オープンループシステムとも言われる。）

(g) Consider the set of *steady states* of this system. The populations in this set change gradually as y increases from 0, and at certain "sufficiently high" levels the set itself changes dramatically. At what level or levels do the "dramatic" changes in the set occur?
定常状態の集合を考えよう。集合の母団数がyのゼロからの増加によって連続的に変動するが、特定な「高い」水準が集合の形が急に変わる。その水準を求めよ。

The "dramatic" change occurs at $y^* = H(Z^*)$. *For* $y < y^*$ *there are two steady states (one unstable, one stable), at* y^* *there is one (unstable) steady state, and for* $y > y^*$ *there are no steady states.*

(h) Solve for the set of steady states, taking care to account for the "dramatic" changes as y increases.
yの増加による集合の形の変化を配慮しながら定常状態集合を求めよ。

For $0 \leq y < y^*$, the steady states are the solutions to the equation $0 = \dot{Z} = H(Z) - y$, or $-bZ^2 + aZ - y = 0$, which has two solutions $\frac{a \pm \sqrt{a^2 - 4by}}{2b}$

At y^* the steady state is Z^*.

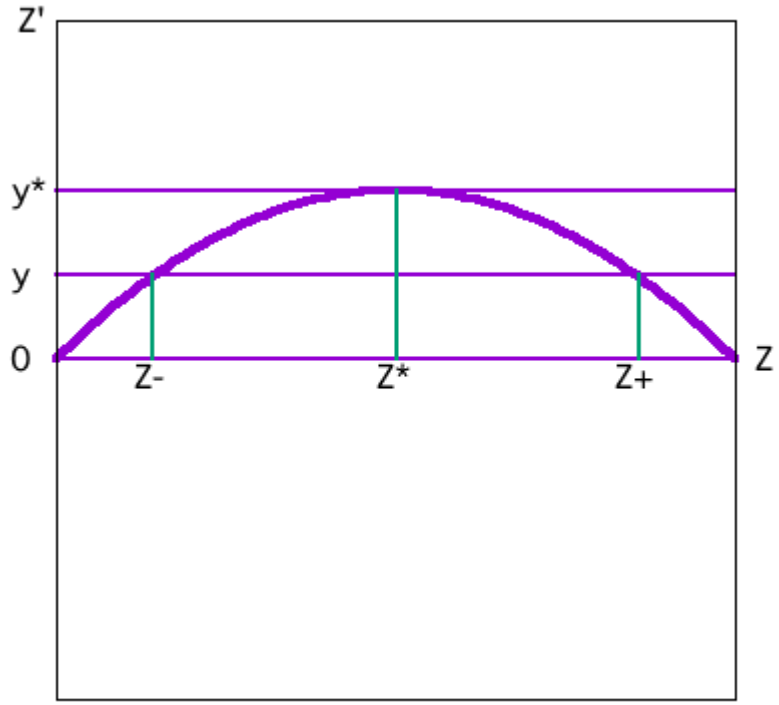
Above y^* the set of steady states is empty.

(i) Which steady states are stable? Which are unstable?
安定定常状態と不安定定常状態を書け。

For $0 \leq y < y^*$, the smaller steady state is unstable, and the larger one is stable.

At y^* the only steady state is unstable.

(j) On the graph below, sketch H and its relation to y. Add labels to indicate the various important parts of the graph. (Don't forget the axes!)
以下のグラフに H と、 y と H との関係をおおよそに描け。注目すべく部分にラベルを振ること。（軸も忘れずに！）



5. `[Problem ID #91] diffeq: solve differential equation`
   Verify that $P(t) = Ae^{\lambda t}$ is a solution to the differential equation $\frac{dP}{dt} = \lambda P$.
   $P(t) = Ae^{\lambda t}$ が微分方程式 $\frac{dP}{dt} = \lambda P$ の解であることを確認せよ。

$$\begin{aligned} \frac{dP}{dt} &= \frac{d}{dt} Ae^{\lambda t} \\ &= \lambda Ae^{\lambda t} = \lambda P. \end{aligned}$$

6. `[Problem ID #5] exhaustible, renewable; definition`
   Describe exhaustible resources, both the general case of *renewable resources* and the special case of *(pure) exhaustible resources*.
   尽くせる資源について述べよ。一般的な再生可能資源と特定の純粋尽くせる資源を含むこと。

   (a) Give a verbal definition of *renewable resource*.
   言葉で再生可能資源の定義を書け。

   A renewable resource is an exhaustible resource that has a bounded positive rate of increase, and thus is self-renewing.

   (b) Give a mathematical description (in terms of variables and functions) of a renewable resource.
   数式（変数・関数など）で再生可能資源の定義を書け。

   Let the stock of the resource be Z. Then the resource stock satisfies the differential equation

$$\dot{Z}(t) = H(Z(t)) - y(t)$$

   for some function H where $y(t)$ is the amount used by society at time t.

   (c) Give a verbal definition of *pure exhaustible resource*.
   言葉で純粋尽くせる資源の定義を書け。

   A pure exhaustible resource is one which can only be used up; its stock can never increase.

   (d) Give the mathematical restriction that characterizes a *pure exhaustible resource* as a special case of a renewable resource.
   再生可能資源の種類として数式（変数・関数など）で純粋尽くせる資源の定義を書け。

   $H(Z) \equiv 0$.

7. `[Problem ID #93] microeconomics; equilibrium, steady state`

   In intermediate economics, *equilibrium* is defined as a state in which all variables take on values so that no agent has an incentive to change behavior. Economic dynamics takes a different approach.

   入門経済学では「均衡」が「全ての変数の値のもとでは行動を変更したい主体がいない」と定義されている。経済動学の観点が違う。

   (a) Define *steady state* as used in dynamics.
   動学で「定常状態」の定義を書け。
   In a steady state, the rate of change of the state variable is zero, so the state does not change.

   (b) Explain why dynamics focuses on steady state rather than equilibrium. (Hint: the Solow model of economic growth may be a good example.)
   動学が均衡ではなくて定常状態を焦点にする理由を説明せよ。
   In equilibrium, no variables will change because all agents are making optimal decisions. In steady state, the state variable does not change, but other variables can change as long as they don't change the state variable.

8. `[Problem ID #23] growth: Solow; basic abstraction: no bond market`
   Some macroeconomic models include the *bond market* as a source of finance for the government deficit. Why does Solow's model omit the bond market?
   マクロ経済学モデルに政府赤字を埋めるために国債を導入する。ソローモデルに国債を無視する理由を説明せよ。
   In the long run the bond market will be in equilibrium, so we can ignore it. Since the bond market does not affect private investment (s is constant) or the labor supply ($L(t)$ is exogenous) in Solow's model, it explains nothing and only adds useless complexity.