Economic Dynamics

Midterm Examination

January 21, 2010

General instructions and marking policy

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** However, if you choose to use Japanese please take great care in writing kanji. Avoid using abbreviated kanji; the only one I know is the 3-stroke mongamae.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited.

Except for calculations, most of the problems can be completely answered within 3 lines. Many questions can be answered within 2 or 3 words. Below each problem ample space is provided. Please write your answers there. Graph paper is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used.

Problems

1. Give the definition of *constant returns to scale*. Show that the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$ satisfies constant returns to scale.

   Constant returns to scale means that if all input factors are changes proportionally, the output will change in the same proportion: $F(\lambda K, \lambda L) = \lambda F(K, L)$. *For the example production function, we have* $10(\lambda K)^{\frac{1}{2}}(\lambda L)^{\frac{1}{2}} = \lambda^{\frac{1}{2}+\frac{1}{2}} \cdot 10K^{\frac{1}{2}}L^{\frac{1}{2}} = \lambda Y$, *so constant returns to scale is satisfied.*

2. Give the definition of *steady state* in dynamics. What is the equation defining steady state in Solow's growth model?

   A steady state is a condition where the state variable (or the vector of state variables) does not change over time. A state variable is an endogenous variable from which all future behavior of the system can be predicted. In the Solow model the state variable is k, *the capital-labor ratio, and the definition of steady state is* $\dot{k} = 0$. *(*$\dot{k} = sf(k) - (n+d)k$ *is the characteristic equation, not the definition.)*

3. The labor force growth rate is *made constant* in Solow's growth model.

   (a) What assumption about the economy is implied by this?

   The important part of the assumption is that labor force growth is exogenous. That is, economic conditions do not affect family planning or the decision to enter the labor force. The fact that it is made constant is not very important, but is related to the fact that we don't yet know very much about why population growth rates differ across countries demography, or even the decision to enter the labor force.

   (b) This assumption is *false* in the real economy. Explain why.

   It is an historical fact that population growth rates rose somewhat with the development of agriculture and the market, rose more in the industrial revolution, rose dramatically with the invention of modern medicine, and then fell again. (These last two facts are known as the "demographic transition.") Many poor countries have not yet completed the demographic transition, so we should expect their population growth rates to change. Also, in modern economies, labor force participation has risen dramatically as women have entered the labor market in large numbers.

(c) Why do you think Solow made this assumption anyway?

Both population dynamics and the labor force entry decision are very hard to model accurately. Furthermore, we know very little about how economic conditions affect those processes. In order to get an usable model, Solow had to leave them out.

4. Consider a Solow model with the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$, a saving rate of $s = 0.2$, a depreciation rate of $d = 0.1$, and a labor force growth rate of $n = 0$.

(a) What is the growth rate of Y in steady state? Explain how you know without solving the model.

The growth rate is $\frac{\dot{Y}}{Y} = n = 0$. *In Solow's model the steady state involves balanced growth of the macro variables at the same rate as the population grows, which we assume is* n.

(b) Suppose the initial population is $L_0 = 1000$. Solve the model, giving expressions for all variables as both macro variables and micro (per worker) values. Don't forget consumption!

The per capita form of the production function is $f(k) = F(k, 1) = 10k^{\frac{1}{2}}$. *The characteristic equation is* $\dot{k} = sf(k) - (n+d)k = 2k^{\frac{1}{2}} - 0.1k$. *Assuming the steady state, we have* $2k^{\frac{1}{2}} = 0.1k$. *Considering this as a quadratic equation in* $k^{\frac{1}{2}}$, *we have two solutions:* $k^{\frac{1}{2}} = 0$ *and* $k^{\frac{1}{2}} = 20$. *That is* $k = 0$ *and* $k = 400$. *The former solution corresponds to an unstable steady state, so we concentrate on the latter,* $k^* = 400$, *which is the stable steady state. Then we have that* $y = 200$ *and* $c = 160$.

For the macro variables, integrating $\dot{L} = nL$ *and using the initial condition gives* $L(t) = L_0e^{nt}$. *The other macro variables are proportional to labor in balanced growth (steady state), so* $K(t) = 400L_0e^{nt}$, $Y(t) = 200L_0e^{nt}$, *and* $C(t) = 160L_0e^{nt}$.

5. Consider the *convergence hypothesis* for the OECD, which *does* show historical evidence for the convergence hypothesis.

(a) Give the convergence hypothesis.

The convergence hypothesis states that the parameters of the Solow model should be quite similar for different countries, and since there is a unique stable steady state, in the long run all economies should converge to that steady state, differing only in population and the other macro variables (which are all proportional to population).

(b) Explain why the convergence hypothesis seems likely to be true.

Technology is knowledge and skill, and all human beings are capable of acquiring that knowledge and skill, so the production function f and the depreciation rate d should be common to all countries. While savings rates and population growth rates vary substantially, the "advanced" countries seem to have similar parameters, so we may expect the emerging market economies to converge there as well.

(c) Draw "before" and "after" graphs of growth rate ($\dot{y}/y$) and income (y, where the unit is current US income in each period) for the OECD.

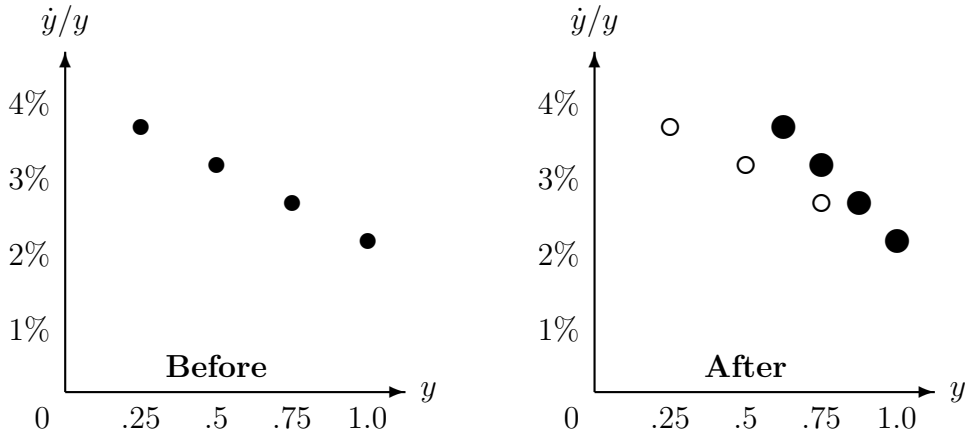

Figure 1: Convergence examples