Economic Dynamics

Makeup Midterm Examination

January 29, 2010

General instructions and marking policy

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** However, if you choose to use Japanese please take great care in writing kanji. Avoid using abbreviated kanji; the only one I know is the 3-stroke mongamae.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited.

Except for calculations, most of the problems can be completely answered within 3 lines. Many questions can be answered within 2 or 3 words. Below each problem ample space is provided. Please write your answers there. Graph paper is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used.

Problems

1. Give the definition of *constant returns to scale*. Show that the production function $Y = 2K + L$ satisfies constant returns to scale.

   Constant returns to scale means that if all input factors are changes proportionally, the output will change in the same proportion: $F(\lambda K, \lambda L) = \lambda F(K, L)$. *For the example production function, we have* $2(\lambda K) + (\lambda L) = \lambda(2K + L) = \lambda Y$, *so constant returns to scale is satisfied.*

2. Give the definition of *steady state* in dynamics. What is state variable in Solow's growth model?

   A steady state is a condition where the state variable (or the vector of state variables) does not change over time. A state variable is an endogenous variable from which all future behavior of the system can be predicted. In the Solow model the state variable is k, *the capital-labor ratio, and the definition of steady state is* $\dot{k} = 0$. *(*$\dot{k} = sf(k) - (n + d)k$ *is the characteristic equation, not the definition.)*

3. The saving rate as a fraction of income is *made constant* in Solow's growth model.

   (a) What assumption about the economy is implied by this?

   The financial markets are always in equilibrium, and money is neutral. That is, any inflation is perfectly even across commodities including labor, and perfectly anticipated. Furthermore, either the interest rate is constant over time (unlikely), or consumers' preferences are such that they balance the (money) value of goods over time, rather than the real quantities of goods.

   (b) This assumption is *false* in the real economy. Explain why.

   We know that consumers and firms react to the interest rate, and the assumption of constant interest rate is neither likely nor historically correct.

   (c) Why do you think Solow made this assumption anyway?

   The only variable that financial markets should affect directly is the saving rate. But we know that this variable is very hard to model accurately. In order to get an usable model, he had to leave it out.

4. Consider a Solow model with the production function $Y = 10K^{\frac{1}{2}}L^{\frac{1}{2}}$, a saving rate of $s = 0.2$, a depreciation rate of $d = 0.08$, and a labor force growth rate of $n = 0.02$.

   (a) What is the growth rate of Y in steady state? Explain how you know without solving the model.

   The growth rate is $\frac{\dot{Y}}{Y} = n = 0.02$. In Solow's model the steady state involves balanced growth of the macro variables at the same rate as the population grows, which we assume is n.

   (b) Suppose the initial population is $L_0 = 1000$. Solve the model, giving expressions for all variables as both macro variables and micro (per worker) values. Don't forget consumption!

   The per capita form of the production function is $f(k) = F(k, 1) = 10k^{\frac{1}{2}}$. The characteristic equation is $\dot{k} = sf(k) - (n+d)k = 2k^{\frac{1}{2}} - 0.1k$. Assuming the steady state, we have $2k^{\frac{1}{2}} = 0.1k$. Considering this as a quadratic equation in $k^{\frac{1}{2}}$, we have two solutions: $k^{\frac{1}{2}} = 0$ and $k^{\frac{1}{2}} = 20$. That is $k = 0$ and $k = 400$. The former solution corresponds to an unstable steady state, so we concentrate on the latter, $k^* = 400$, which is the stable steady state. Then we have that $y = 200$ and $c = 160$.

   For the macro variables, integrating $\dot{L} = nL$ and using the initial condition gives $L(t) = L_0e^{nt}$. The other macro variables are proportional to labor in balanced growth (steady state), so $K(t) = 400L_0e^{nt}$, $Y(t) = 200L_0e^{nt}$, and $C(t) = 160L_0e^{nt}$.

5. Understanding the role of technological progress in economic growth is very important.

   (a) In economics, technological progress is represented by an increase in output from constant factor inputs. There are three special representations of technological progress: *Harrod neutral technological progress*, which increases the productivity of labor, *Solow neutral technological progress*, which increases the productivity of capital, and *Hicks neutral technological progress*, which increases the productivity of both factors by the same amount. Which is most useful in adapting Solow's model to include technological progress? Explain briefly.

   Harrod neutral technological progress is used to extend the Solow model of growth to account for technological progress in theoretical analysis. It has the special property that we can define "effective labor units" which includes a productivity factor as well as "clock" hours of labor, and define the state variable as $\tilde{k}$ which is the ratio of capital to effective labor AL. If the productivity factor A grows over time as $A(t) = A_0 e^{\lambda t}$, then the mathematical analysis of the steady state for $\tilde{k}$ is the same as that used for the original Solow model.

   The other two kinds of technological progress do not admit such a simple analysis.

   (b) What is the *characteristic equation* for the Solow model with technological progress?

   $\dot{\tilde{k}} = sf(\tilde{k}) - (n + d + \lambda)\tilde{k}$.

   (c) In this model, although the capital to effective labor ratio is constant in steady state, and so output per effective labor is constant, the consumption per worker increases steadily. Explain.

   $\tilde{y}^* = \frac{Y}{AL} = \frac{1}{A} y^*$, *so* $y^* = A_0 e^{\lambda t} \tilde{y}^*$. *Since the second factor* $e^{\lambda t}$ *is growing and the other two are constant,* y^* *grows.*