Economic Dynamics / 経済動学

Final Examination / 期末試験

December 21, 2018 / 平成３０年１２月２１日

General instructions / 一般説明

If the English and Japanese versions of any text differ in meaning, the English text is more reliable. However, please ask for clarification if you have any doubt.

各文書の英文と和文の間に食い違いがあれば、英語の方に信頼をおいた方がよい。ただし、不明な点については遠慮なく聞いてください。

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** In Japanese, please take great care in writing kanji. Avoid abbreviated kanji; the only one I know is the 3-stroke mongamae. Also try to use *Japanese* kanji; Chinese hanzi and Korean hanja are often not recognizable to me.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited.

Except for calculations, most of the problems can be completely answered within 3 lines. Many questions can be answered within 2 or 3 words. Below each problem ample space is provided. Please write your answers there. Graph paper is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used.

名前と学籍番号を忘れずに各ページに記入してください。

以下にマクロ経済学についての問題のすべてに解答せよ。**解答の言語は日本語でも英語でも構わない。**もし日本語で書けば漢字などの書き方に十分注意してください。たとえ、省略した漢字などを使わないで。また、日本の「漢字」を使って下さい。中国と韓国の形は分からない場合が多いです。

（私が読めない場合には省略した文字を「間違え」と採点します。）

ノート・教科書・辞書・電卓・携帯電話・その他のメモリを持つ電子製品の使用は禁止である。全ての計算は簡単であるので電卓などは必要ない。

机の上にペン・鉛筆・消しゴム・鉛筆削り・時計・この試験用紙の他の物を置かないこと。その他のものを側の席に置くこと。

後ろの面を使ってもよい。

Problems / 問題

Each question is worth 10 points, unless otherwise specified.

1. `[Problem ID #70] diffeq: IFS; recursive-to-difference`
   Convert the recursive dynamic model $x_{t+1} = f(x_t)$ to difference equation form.

   $\Delta x_t \equiv x_{t+1} - x_t = f(x_t) - x_t \equiv g(x_t)$.

2. `[Problem ID #27] growth: Solow; phase diagram solution`
   The graph in Fig. 1 is used to analyze Solow's growth model.

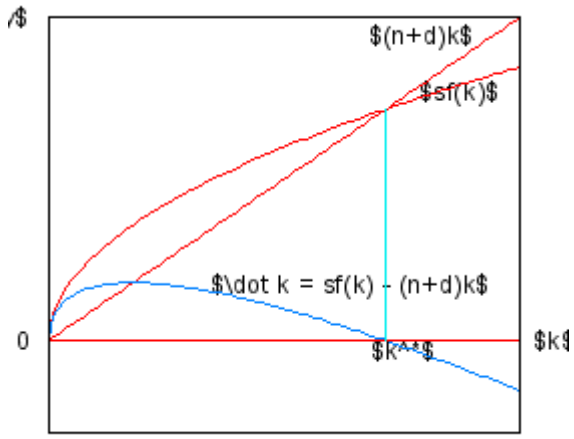

Figure 1: Solution

   (a) Label the axes, important intersection points, and curves with appropriate variables and functions.
   The labels for the production function conditions on derivatives are unnecessary, but will get extra credit.

   (b) Find k^* and plot it on the graph.

   (c) Sketch the curve representing $\dot{k}$ as a function of k. Overall, the curve can be quite approximate, but intersections defining steady states must be accurate.

   (d) If k starts at $k0$ now, what can you say about its future behavior?
   k will increase over time, eventually converging to k^* unless disturbed by an outside force.

3. `[Problem ID #50] exhaustible: renewable; natural rate of increase`

   The natural rate of increase of a population $\dot{Z}$ which is not being harvested (*i.e.*, $y = 0$) is described by a function $H(Z)$, leading to the differential equation $\dot{Z} = H(Z)$.

(a) The logistic form $H(Z) = \beta Z(1 - \delta Z)$ is often used to describe a *renewable resource* (population) whose growth is constrained by some scarce resource. Explain how this mathematical form expresses that constraint.

When the population is near $\bar{Z} = 1/\delta$ the last factor is approximately 0, and $H(Z) = 0$ is a good approximation. For $Z < \bar{Z}$, $H(Z) > 0$, so as $Z \to \bar{Z}^-$, growth slows and eventually stops. The population has run out of the resources needed for growth, and can barely sustain itself.

(b) Give a formula for the H function that describes a *pure exhaustible resource* like oil.

$H(Z) = 0$.

(c) The formula for the H function for a "renewable resource" like solar power would be the same as for oil. This seems to be a paradox. Explain briefly how this can be. (Hint: how does consumption y affect the "stock" of solar power?)

Unlike resources such as oil and fish, for solar power the "stock" and the "flow" are different. The stock is the fusible gases (mostly hydrogen) in the Sun, the flow is light energy. Using light energy does not affect the Sun's ability to produce light, so the amount of light reaching Earth is a constant $\bar{y}$, unaffected by past consumption $y(t)$.

4. `[Problem ID #49] production function; neoclassical conditions`

There are three conditions that must be satisfied by the per-capita production function $f(k)$. For each condition:

- Give its name or explain its meaning in words.
- Give an equation or inequality defining the condition precisely.
- Show that the function $f(k) = k^{\frac{1}{2}}$ satisfies the condition.

The function must require input*, that is $f(0) = 0$.*

The function must be productive*, that is $f'(k) > 0$ for all $k \geq 0$.*

The function must display diminishing marginal returns to capital*, that is $f''(k) < 0$ for all $k \geq 0$.*

The Inada conditions on slopes at zero and infinity are technical conditions, convenient in some analyses such as existence of stable steady state in the Solow model, but not part of the definition.

5. `[Problem ID #97] innovation: value function`

Boldrin and Levine model the spread of inventions by assuming that it takes

time to buy a copy of a new invention and then make and sell further copies. They solve their model by defining a series of functions

$$v_t(k) = \max_c u(c) + \delta v_{t+1}(\beta(k-c) + \zeta c).$$

(a) What do we call a function like $v_t(k)$?

$v_t(k)$ *is a* value function*, giving the optimal value to acting in state* k*.*

(b) Give the equation that defines a *stationary solution* to the problem.

$v_t(k) = v_{t+1}(k) = v(k)$ *for all* t *and* k*.*
Alternatively, substituting v *for* v_t *and* v_{t+1}*,* $v(k) = \max_c u(c) + \delta v(\beta(k-c) + \zeta c)$*.*

(c) Explain the meaning of the equation $p_t = u'(c_t)$ in economic terms.

This is the usual first order condition of the consumer's static optimization, price = marginal utility (with the marginal utility of money = 1).

(d) Explain the meaning of the equation $q_t = v_t'(k_t)$ in economic terms.

This is the usual first order condition of the consumer's dynamic optimization, price of asset = marginal present discounted value from holding more of the asset (with the marginal utility of money = 1).

(e) What equation characterizes market equilibrium in period t?

$p_t = q_t$*.*
This holds because it's not possible to determine whether a customer is a consumer or a reseller or both, so both must receive the same price. This says that the marginal unit is just as valuable when consumed as when held as a productive asset for its future value. In the Boldrin-Levine model with simultaneous copying and consumption, "production" is the incremental value of not consuming. i.e.*, the extra copying from not consuming.*

6. `[Problem ID #98] chaos: characterization`
Characterize each of the following properties that a chaotic dynamic process might have as *true* or *not true*. For those that are *not* true, explain why not. (The majority of credit depends on your explanation of *why not*. A correct explanation can be a short as one brief sentence for each answer, but you may use as many words as you need.)
カオスプロセスの特徴として考えられる、以下のことについて「ある」か「ない」と答えた上、ないの場合には説明せよ。

A chaotic dynamic process:
カオスプロセスは

(a) cannot be predicted accurately in the (sufficiently) far future.
長い将来なら正確に予測できない。

True.

More precisely, unless it is possible to measure real numbers infinitely accurately, calculation errors accumulately rapidly.

(b) has no fixed points or cycles.
サイクルがない。

True.

(c) is random.
ランダムである。

False.

A chaotic process is deterministic: the next value is calculated from the current one. If you know a chaotic process's function, you can always calculate the next value as precisely as you can measure. In a random process, there is always a substantial error possible, characterized by the standard deviation.

(d) will fill the entire space between its maximum and minimum limits, if you try enough different starting points.
初期の値を変動させれば最低値と最高値の間のすべてのポイントに当 たる。

False.

Many chaotic processes converge to so-called strange attractors*, which are subsets of the space. (Of course if you start from every point you can fill the space but that doesn't have anything to do with the chaotic process.)*

7. `[Problem ID #102] innovation: spillover and market failure`

"Spillovers" of technological progress (innovation) from innovators to other actors in society are believed to create a *market failure*. Describe this market failure.

It is the unpriced benefits from spillovers of innovation (knowledge of possibilities, reverse engineering, etc*., preventing the innovator from recovering the full value of the innovation. If the innovator estimates the cost of innovation to be greater than the value to her, she may choose to suppress a socially valuable innovation.*

8. `[Problem ID #106] innovation: vs. invention`
Economists typically differentiate between *invention* and *innovation*.
経済学では「発明」と「技術革新」を区別する。

(a) Explain the difference between these two social processes.
その違いを説明せよ。

Both invention and innovation make an activity possible. The difference is that invention only makes it possible for the inventor, while an innovation based on that invention either provides new products for the market, or makes existing products cheaper.

(b) Explain why this difference is important in economics.
経済学ではその違いの重要な意義を説明せよ。

An invention has negligible economic effects (it makes new activity possible only for the inventor), while an innovation makes it possible for the whole world (in principle) through the market.

(c) Explain why *innovation* is by definition a dynamic process.
技術革新があくまで動学的であることの理由を説明せよ。

An innovation changes the constraints in the economy, as activities that were impossible (viewing a new painting, producing electricity without pollution) become possible and subject to economic choice.

9. `[Problem ID #innovation: second best] 103`
According to the "theory of the second best," combatting the market failure associated with technological progress (innovation) may require creating an additional market failure. Name the policies most countries use to repair the first market failure, and explain how they result in the second market failure.
セコンドベスト理論により、技術進歩 による市場の失敗を直すには他の市場の失敗を認めなければならない場合がある。前者の失敗に対応するためのよく使われた対策とその後者への効果を説明せよ。

It not necessary to mention the first market failure, but of course it is the unpriced benefits from spillovers of innovation (knowledge of possibilities, reverse engineering, etc*., preventing the innovator from recovering the full value of the innovation.*

The policies used attempt to approximately capture the value of these benefits by creating copyrights to protect expressive innovations and patent to protect productive innovations. The second market failure is the creation of monopolies, which result in the usual static inefficiencies (price > marginal cost), and further may work to inhibit future innovation.

10. `[Problem ID #109] OLG: dynamic structure`
Describe the dynamic structure of the overlapping generations model explained in class.
広義で説明された overlapping generations model の動学的構造を述べろ。

- What is the *life cycle* of an individual member of a generation?
世代の各個人のライフサイクルを説明せよ。

  Each generation lives for two periods, "young" and "old". When young, the agent works productively and receives monetary compensation, but does not consume. When old, the agent does not work and uses money to buy the consumption good.

- What is the generational structure of the population in each period?
  世代による人口の構造を説明せよ。
  In each period, the population is half-young, half-old.