Economic Dynamics / 経済動学

Final Examination / 期末試験

December 25, 2020 / 令和 ２年１２月２５日

General instructions / 一般説明

If the English and Japanese versions of any text differ in meaning, the English text is more reliable. However, please ask for clarification if you have any doubt.

各文書の英文と和文の間に食い違いがあれば、英語の方に信頼をおいた方がよい。ただし、不明な点については遠慮なく聞いてください。

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** In Japanese, please take great care in writing kanji. Avoid abbreviated kanji; the only one I know is the 3-stroke mongamae. Also try to use *Japanese* kanji; Chinese hanzi and Korean hanja are often not recognizable to me.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited.

Except for calculations, most of the problems can be completely answered within 3 lines. Many questions can be answered within 2 or 3 words. Below each problem ample space is provided. Please write your answers there. Graph paper is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used.

名前と学籍番号を忘れずに各ページに記入してください。

以下にマクロ経済学についての問題のすべてに解答せよ。**解答の言語は日本語でも英語でも構わない。**もし日本語で書けば漢字などの書き方に十分注意してください。たとえ、省略した漢字などを使わないで。また、日本の「漢字」を使って下さい。中国と韓国の形は分からない場合が多いです。

（私が読めない場合には省略した文字を「間違え」と採点します。）

ノート・教科書・辞書・電卓・携帯電話・その他のメモリを持つ電子製品の使用は禁止である。全ての計算は簡単であるので電卓などは必要ない。

机の上にペン・鉛筆・消しゴム・鉛筆削り・時計・この試験用紙の他の物を置かないこと。その他のものを側の席に置くこと。

後ろの面を使ってもよい。

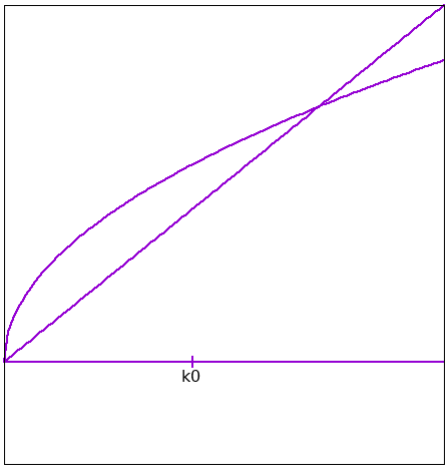
This page contains no test content.
試験内容のないのページです。

Problems / 問題

Each question is worth 10 points, unless otherwise specified.

1. The graph in Fig. 1 is used to analyze Solow's growth model.

Figure 1: Curves used in Solow model



   (a) Label the axes, important intersection points, and curves with appropriate variables and functions.

   (b) Find k^* and plot it on the graph.

   (c) Sketch the curve representing $\dot{k}$ as a function of k. Overall, the curve can be quite approximate, but intersections defining steady states must be accurate.

   (d) If k starts at $k0$ now, what can you say about its future behavior?

2. Convert the dynamic model in difference equation form $\Delta x_t = g(x_t)$ to recursive equation form.

3. Describe the dynamic structure of the overlapping generations model explained in class.
広義で説明された overlapping generations model の動学的構造を述べろ。

   - What is the *life cycle* of an individual member of a generation?
世代の各個人のライフサイクルを説明せよ。

   - What is the generational structure of the population in each period?
世代による人口の構造を説明せよ。

4. The natural rate of increase of a population $\dot{Z}$ which is not being harvested (*i.e.*, $y = 0$) is described by a function $H(Z)$, leading to the differential equation $\dot{Z} = H(Z)$.

   (a) The logistic form $H(Z) = \beta Z(1 - \delta Z)$ is often used to describe a *renewable resource* (population) whose growth is constrained by some scarce resource. Explain how this mathematical form expresses that constraint.

   (b) Give a formula for the H function that describes a *pure exhaustible resource* like oil.

   (c) The formula for the H function for a ”renewable resource” like solar power would be the same as for oil. This seems to be a paradox. Explain briefly how this can be. (Hint: how does consumption y affect the ”stock” of solar power?)

5. In the Solow model, there are three conditions that must be satisfied by the per-capita production function $f(k)$. For each condition:

   - Give its name or explain its meaning in words.
   - Give an equation or inequality defining the condition precisely.
   - Show that the function $f(k) = k^{\frac{1}{2}}$ satisfies the condition.

6. Describe exhaustible resources, both the general case of *renewable resources* and the special case of *(pure) exhaustible resources*.
   尽くせる資源について述べよ。一般的な再生可能資源と特定の純粋尽くせる資源を含むこと。

   (a) Give a verbal definition of *renewable resource*.
   言葉で再生可能資源の定義を書け。

   (b) Give a mathematical description (in terms of variables and functions) of a renewable resource.
   数式（変数・関数など）で再生可能資源の定義を書け。

   (c) Give a verbal definition of *pure exhaustible resource*.
   言葉で純粋尽くせる資源の定義を書け。

   (d) Give the mathematical restriction that characterizes a *pure exhaustible resource* as a special case of a renewable resource.
   再生可能資源の種類として数式（変数・関数など）で純粋尽くせる資源の定義を書け。

7. Consider the case of the lines outside restaurants.

   (a) Explain the idea of an *information cascade*.

   (b) When a diner uses the length of lines to help decide which restaurant to choose, what information is she inferring from the lines?

   (c) Why does she trust that information?

8. **(20 points)** We discussed the *fishery* model of a single population as renewable resource with constraints on growth. Consider the *logistic model* of population growth:

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z).$$

漁業での魚母団の成長制限された再生可能資源としたモデルを講義で触 れた。母団成長のロジスティックモデルを検討しよう：

$$\dot{Z} = H(Z) = aZ(1 - \frac{b}{a}Z)$$

   (a) What do the expressions a and bZ represent? (Hint: it may be useful to expand the expression given for $H(Z)$.)
   a と bZ という式は何を表現するか。（ヒント： $H(Z)$ の別の式が役立つかも。）

(b) What can you say about the case $b = 0$?
$b = 0$ の場合には何が言えるか。

(c) Show that H satisfies the four conditions for a *bell curve*.
H がベル曲線の 4 条件を満たすことを証明せよ。

(d) Solve for the populations $\underline{Z}$ and $\bar{Z}$ such that $H(\underline{Z}) = H(\bar{Z}) = 0$.
母団数 $\underline{Z}$ と $\bar{Z}$ は $H(\underline{Z}) = H(\bar{Z}) = 0$ を満たす。 $\underline{Z}$ と $\bar{Z}$ の解を求めよ。

(e) What is special about the populations $\underline{Z}$ and $\bar{Z}$?
母団数 $\underline{Z}$ と $\bar{Z}$ の特徴を説明せよ。

(f) Solve for the population Z^* such that $H'(Z^*) = 0$.
$H'(Z^*) = 0$ を満たす母団数 Z^* の解を求めよ。

9. *Power laws* are an important phenomenon in the economics of the World Wide Web and e-commerce.

   (a) What is a "power law"?

   (b) What is the *Central Limit Theorem*?

   (c) Give at least two ways in which a power law is non-normal.

   (d) Among those in Part 9c, what is the economically most important deviation from normality of a power law? Explain why you chose it.