Economic Dynamics

SAMPLE Examination Questions and Answers

June 2005

General instructions and marking policy

Note: this is not a representative sample. These questions are all possible, but there are many others. You should not conclude anything about the relative likelihood of question from this sample.

Several problems in Economic Dynamics are presented below. **You may answer in Japanese or English.** However, if you choose to use Japanese please take great care in writing kanji. Avoid using abbreviated kanji; the only one I know is the 3-stroke mongamae.

Use of notes, textbooks, dictionaries, and so on is prohibited. All calculations are simple, so the use of calculators is also prohibited.

Except for calculations, most of the problems can be answered within 3 lines. Some questions can be answered with only a few words. Below each problem ample space is provided. Please write your answers there. Graph paper is provided for graph problems. Please use it. In calculations, in addition to the result itself, please also write any equations used.

Problems

1. `[Problem ID #IDNO] durable goods monopoly`
   Explain how a monopolist selling a *durable good* faces "competition with himself."

2. `[Problem ID #IDNO] fishery; LR equilibrium, phase diagram`
   In the case of a fishery with X firms (boats), average productivity $f(Z, X)$, positive cost of extraction p, and price for consumption q, we saw that we could use the zero-profit condition for long-run equilibrium to give the equation $p = qf(Z, X)$, and that we could solve to get the long-run number of firms in the industry as $X = M(Z)$.

   (a) Give an expression for the total harvest Y in long-run equilibrium as a function of Z.

   (b) On the graphs below, draw the law of natural increase $H(Z)$ and the equilibrium harvest function $Y(Z)$ from part 2a. On the left graph, show the case with a stable steady state. On the right graph show the case where population collapses to zero due to overharvesting.

3. `[Problem ID #IDNO] renewable; law of increase`
   Let $H(Z)$ be the law of increase for a population Z. Explain the economic meaning of the *own rate of return* $H'(Z) = \lim_{\delta \to 0} \frac{H(Z+\delta)-H(Z)}{\delta}$.

4. `[Problem ID #IDNO] exhaustible; backstop technology, choke price`

   Explain the relationship between a *backstop technology* such as solar power and the possibility of a *choke price* for an exhaustible resource such as oil. Be sure to include definitions of both terms.

5. `[Problem ID #IDNO] exhaustible, renewable; definition`
   Describe exhaustible resources, both the general case of *renewable resources* and the special case of *(pure) exhaustible resources.*

(a) Give a verbal definition of *renewable resource.*

(b) Give a mathematical description (in terms of variables and functions) of a renewable resource.

(c) Give the restrictions on values, slopes, *etc.*, that characterize the typical "bell-shaped" function for "natural increase" of a renewable resource.

(d) Give a verbal definition of *pure exhaustible resource.*

(e) Give the mathematical restrictions (in addition to those of part 5c) that characterize a *pure exhaustible resource.*

6. `[Problem ID #IDNO] exhaustible; price path with extraction cost`

Consider an exhaustible resource with an extraction cost function

$$C(R, S, t) = f(t)Rg(R)G(S)$$

where R is rate of extraction, S is current stock, f is cost decreases from improved technology ($f' < 0$), $g(R)$ is decreasing returns to scale ($g' \geq 0$) and $G(S)$ is increased efficiency of extracting from a larger stock ($G' \leq 0$). The equilibrium condition is

$$\frac{\dot{p}_t}{p_t} = r + \frac{f(t)R_t g(R_t)G'(S_t)}{p_t}$$

. Briefly describe the price path of the resource, comparing it to a resource that can be extracted costlessly, in the short and long run.

7. `[Problem ID #IDNO] exhaustible; rational bubble`
Discuss the possibility of a "rational bubble" in the market for an exhaustible resource.

(a) Describe the price path, the initial price, and the long run behavior of the stock of the resource.

(b) Explain how and why a "bubble" deviates from efficiency.

(c) Explain why market pressure may not restore efficiency.

8. `[Problem ID #IDNO] asset arbitrage; Hotelling Rule`
Following steps (a)–(c), derive the *Hotelling Rule* from the arbitrage condition that the markets for both bonds and the stock of an exhaustible resource be in equilibrium. Also, answer (d).

    (a) Formulate the *arbitrage equation* that defines asset market equilibrium in terms of the interest rate, current price of the exhaustible resource, and future price of the exhaustible resource, for a short period of time.

    (b) Explain why markets that don't satisfy this condition are not in equilibrium.

    (c) Show that by manipulating the equation and taking the limit as the period that the assets are held goes to zero, the Hotelling Rule for the price of the exhaustible resource can be expressed as an equation involving the growth rate of price and the interest rate.

    (d) Explain why the Hotelling Rule implies no steady state for the exhaustible resource.

9. `[Problem ID #IDNO] value function; formula`
Give a formula defining the *value function* used in backward induction analysis.

There are two reasonable ways to define the value function. One is direct, the present value of always making the optimal decision for the rest of time:

$$V_t^*(X_{t-1}) = \sum_{\tau=t}^{T} U(s_\tau^*(X_{\tau-1})),$$

where $X_\tau = X_{\tau-1} + s_\tau$. *The second is recursive, defining the value function in terms of the next period's value function:*

$$V_t^*(X_{t-1}) = U(s_t^*(X_{t-1})) + V_{t+1}^*(X_t),$$

where $X_t = X_{t-1} + s_t$.

10. `[Problem ID #IDNO] value function; definition, use`
Suppose the formula defining the function $Z()$ is

$$Z_t(X_{t-1}) = \max_{s_t} U(s_t(X_{t-1})) + Z_{t+1}(X_t).$$

What is Z? How is it used in dynamic analysis?

11. `[Problem ID #IDNO] backward induction; computation`
Fill in the following table for backward induction. The starting balance is 0, the target balance at the end of three years is 15000. Utility is logarithmic ($u(C) = \ln C$). The discount factor is $\delta = 1$ in all periods. *You do not need to evaluate the utility function; leave it in the form "*$\ln 10000$*" when consumption is 10000, for example. Remember that the logarithmic utility function tries to make consumption equal in each period, if possible.*

12. `[Problem ID #IDNO] backward induction; tree cutting`
Consider a company which grows trees and cuts them for sale as lumber. It can only grow one tree at a time. The value of the tree increases according to the function $R(\tau) = R_1\tau^2$. When it cuts the tree, it sells it, then plants another tree of value 0, which starts to grow.

    Assume that the tree can only be harvested at a particular date each year, and that the company is planning for three years. Thus, the tree must be cut at $t = 3$ (and the model ends), and the tree may be cut at $t = 1$ or at $t = 2$, in which case another tree is planted and the problem continues until $t = 3$. The company wishes to maximize the total revenue from selling the tree(s).

    In mathematical terms, let the amount the company receives for selling a tree at time t be S_t. It wishes to maximize $S_1 + S_2 + S_3$. Solve the problem by backward induction.

Year (t)	**1**	**2**	**3**
Income (Y)	20000	20000	20000
Saving ($S = s_t^*(X_{t-1})$)	5000	5000	5000
Consumption (C)	15000	15000	15000
Current utility ($u(C)$)	$\ln 15000$	$\ln 15000$	$\ln 15000$
Ending balance (X)	5000	10000	15000
Future value ($V_t = V_t^*(X_{t-1})$)	$2 \ln 15000$	$\ln 15000$	0
Optimal saving ($s_t^*(X_{t-1})$)	$\dfrac{15000 - X_0}{3}$	$\dfrac{15000 - X_1}{2}$	$15000 - X_2$
Value function ($V_t^*(X_{t-1})$)	$2 \ln \left(\dfrac{25000 + X_0}{2} \right)$	$\ln(5000 + X_1)$	0

- Let the state variable at time t be A_t, the *age* of the currently growing tree. *E.g.*, if the tree was last cut at time $t = 1$, and "now" is $t = 3$, $A_3 = 2$. Explain why this is a good choice of state variable.

  Since tree size depends only on the age of the tree, and the revenues only depend on tree size when cut, the age of the current tree gives all the information you need. You don't need to know how big any of the previous trees were when they were cut, for example.

- Compute the optimal revenue for time $t = 3$ as an expression in terms of A_3 (*i.e.*, $S_3 = 0$ if the tree is not cut, or $S_3 = R(A_3)$ if the tree is cut and it is A_3 years old).

  This is a "trick" question. The problem says you have to cut the tree, therefore $S_3^*(A_3) = R(A_3) = R_1 A_3^2$.

- Compute the value function $V_3(A_3)$ as an expression in terms of s_3. [*This should be* A_3*!*]

  $V_3(A_3) = S_3^*(A_3) = R_1 A_3^2$.

- Compute S_1, V_2, and S_2 by maximizing $S_t + V_{t+1}(A_{t+1})$ at times $t = 2$ and then $t = 1$.

  $S_2^*(A_2)$ *must maximize* $S_2 + V_3(A_3)$. *If you cut the tree, then* $S_2 = R(A_2)$ *and* $A_3 = 1$. *Thus total revenue is* $S_2 + V_3 = R_1 A_2^2 + V_3(1) = R_1(A_2^2 + 1)$. *If you do not cut the tree, then* $S_2 = 0$ *and* $A_3 = A_2 + 1$. *Thus* $S_2 + V_3 = 0 + V_3(A_2 + 1) = R_1(A_2^2 + 2A_2 + 1)$. $2A_2 > 0$, *so you should never cut the tree at* $t = 2$. *Thus* $S_2^*(A_2) = 0$ *no matter what* A_2 *is!*

  Then $V_2(A_2) = S_2^*(A_2) + V_3(A_2 + 1) = R_1(A_2^2 + 2A_2 + 1)$.

  Similar logic shows that $S_1^*(A_1) = 0$, *and* $V_1(A_1) = V_2(A_1 + 1) = V_3(A_1 + 2) = R_1(A_1^2 + 4A_1 + 4)$.

- Compute the optimal tree-cutting plan for all three years.

  $A_1 = 1$, *so we never cut the tree until* $t = 3$, $S_1 = S_2 = 0$, *and the total revenue is* $S_3 = 9R_1$.

- Describe the meaning of $V_2(A)$ in words.

  This is the expected total revenue from having a tree A *years old at time* $t = 2$, *assuming you choose optimally at time* $t = 3$.

Note: You may notice that the notation $V_t(A_t)$ (both subscripts are t) is different from that used in class, $V_{t+1}(X_t)$ (one is $t + 1$, the other t). This is correct and more convenient for this problem.

13. `[Problem ID #IDNO] dynamics; classifying cases`
    A company is growing trees to sell as materials to make wood products.

The longer a tree grows, the more wood it contains, and the more revenue it brings. The company must decide when to cut and sell each tree. Is this a dynamic problem? Explain why or why not.

14. `[Problem ID #IDNO] dynamics; classifying cases`
    Consider the following two situations involving time. One is dynamic and the other is not.

    - **Case A:** An oil-exporting country decides to try to increase oil production by 10% per year for many years to come.
    - **Case B:** A country that makes computer memory chips decides to try to increase chip production by 10% per year for many years to come.

    Which is dynamic, Case A or Case B? Explain why you chose the one you say is dynamic. Explain why the other one is not dynamic.

    Case A is dynamic because the increases from year to year are limited by the stock of oil; once sold, it is gone and the stock irreversibly decreases to zero. Case B is not dynamic because there is no influence of one year's production on the next year.

15. `[Problem ID #IDNO] dynamics; classifying cases`
    A farmer is raising tomatoes. It takes about 10 weeks to raise tomatoes. They must be harvested just before they are ripe, because they continue to get ripe while being shipped. So depending on the distance to the market, the farmer will harvest a little earlier or later. Is this a *dynamic* problem? Explain why or why not.

16. `[Problem ID #IDNO] dynamics; classifying cases`
    Student A finds that he cannot take Economic Dynamics because Macroeconomics, a course required for graduation, is taught at the same time. Student B finds that she cannot take Economics Dynamics because she has not completed Microeconomics, which is set as a prerequisite for Economic dynamics. Which student faces the more *dynamic* constraint? Explain your choice.

    It is possible to argue that Student A's situation is more dynamic. However, I think that Student B faces a more dynamic choice, because when the decision to take Microeconomics or not is being considered, she must also take into account the fact that a future choice, whether or not to *take Economic Dynamics, is affected by the choice to take Microeconomics. That is, choices of actions at different times affect each other.*

17. `[Problem ID #IDNO] dynamics; irreversibility, example`
Give an example of an *irreversible* process in the economy, and explain the relationship of irreversibility to economic dynamics.

18. `[Problem ID #IDNO] dynamics; modeling`
Consider a student's total number of credits earned as a *state variable.*

    (a) Explain how this state variable helps you in dynamically planning your academic career (*e.g.*, choice of courses).

    (b) Do you think a plan that says, "when you have X credits, take courses A, B, and C," is a sensible plan? Explain why or why not.

19. `[Problem ID #IDNO] dynamics; modeling`
Consider a student's *grade point average* (GPA) as a *state variable.* GPA first converts the grade earned in each course to *grade points* (GP) using the scale A = 4, B = 3, C = 2, D = 1, F = 0. Then each grade is weighted by the number of credits earned for the course. Finally, the sum of weighted GP is divided by the total number of credits:
$$\text{GPA} = \frac{\sum_{i=1}^{n} \text{credits}_i GP(\text{grade}_i)}{\sum_{i=1}^{n} \text{credits}_i}.$$

    (a) Explain how this state variable helps you in dynamically planning your academic career (*e.g.*, choice of courses).

    (b) Do you think a plan that says, "when you have a GPA of X, take courses A, B, and C," is a sensible plan? Explain why or why not.

20. `[Problem ID #IDNO] microeconomics; scarcity definition`
What is a *scarce resource* in microeconomics? Be sure to define both "scarce" and "resource."

    A scarce resource is something that is useful (a resource*), that somebody could benefit from using more of it, but there isn't any more free to use (it is* scarce*).*

21. `[Problem ID #IDNO] microeconomics; allocation definition`
    What is *allocation* in microeconomics? How is allocation related to *opportunity cost*?

    Allocation *is the decision to use a particular scarce resource to achieve one goal, at the expense of other goals. The* opportunity cost *of an decision is measured by the goals that were sacrificed, rather than the resources that were used. Allocation emphasizes the relation of decisions about resources to goals, so opportunity cost is the right way to measure cost in allocation problems.*

22. `[Problem ID #IDNO] technology; R&D racing, overproduction`
    What is the socially undesirable result of "R&D racing," for example to be the first to patent a new medical drug?

23. `[Problem ID #IDNO] Solow model; basic abstraction`
    Some macroeconomic models include the *bond market* as a source of finance for the government debt. Why does Solow's model omit the bond market?

    In the long run the bond market will be in equilibrium, so we can ignore it. Since the bond market does not affect private investment (s is constant) or the labor supply ($L(t)$ is exogenous) in Solow's model, it explains nothing and only adds useless complexity.

24. `[Problem ID #IDNO] Solow model; basic abstractions`
    The money supply is *not* considered in Solow's model.

    (a) What assumption about the economy is implied by this?

    (b) Why do you think Solow left this variable out of his model?

25. `[Problem ID #IDNO] Solow model; basic abstractions`
    The money supply is *not* considered in Solow's model.

    (a) What assumption about the economy is implied by this?

    (b) Why do you think Solow left this variable out of his model?

26. `[Problem ID #IDNO] Solow model; notation`
Consider *Solow's growth model.* Write an expression expressing the "rate of saving per worker."

27. `[Problem ID #IDNO] Solow model; steady state definition`
Consider *Solow's growth model.* Write an equation *defining* the steady state.

This is trivial: $\dot{k} = 0$.

28. `[Problem ID #IDNO] Solow model; phase diagram solution`
The following graph is used to analyze Solow's growth model.

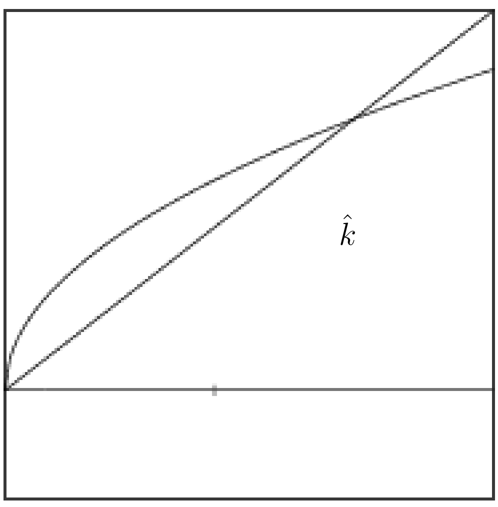
    (a) Label the axes, important intersection points, and curves with appropriate variables and functions.

        The labels for the production function conditions on derivatives are unnecessary, but will get extra credit.

    (b) Find k^* and plot it on the graph.

    (c) Sketch the curve representing $\dot{k}$ as a function of k. Overall, the curve can be quite approximate, but intersections defining steady states must be accurate.

    (d) If k starts at $\hat{k}$ now, what can you say about its future behavior?



29. `[Problem ID #IDNO] Solow model; phase diagram solution identical to previous (up to graph)`
The following graph is used to analyze Solow's growth model.

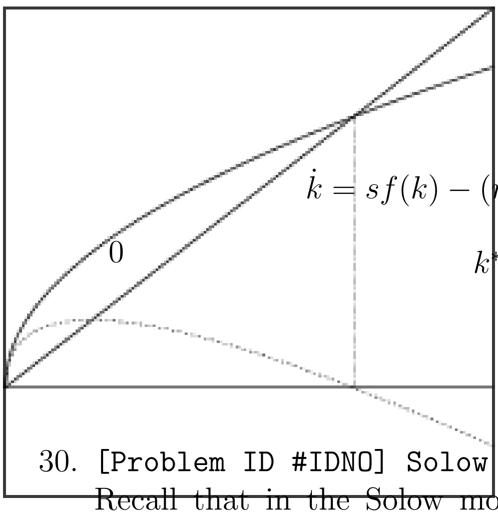
(a) Label the axes, important intersection points, and curves with appropriate variables and functions.

The labels for the production function conditions on derivatives are unnecessary, but will get extra credit.

(b) Find k^* and plot it on the graph.

(c) Give the simplest possible expression describing $\dot{k}$ in *steady state balanced growth.*

"Trick" question. By definition, $\dot{k} = 0$*.*



30. `[Problem ID #IDNO] Solow model; neutral progress`
Recall that in the Solow model with (Harrod-neutral) technological progress, the characteristic equation is $\dot{k} = sf(k) - (n + d + \lambda)k$. Name and briefly describe each of the following symbols from the characteristic equation:

(a) k

(b) s

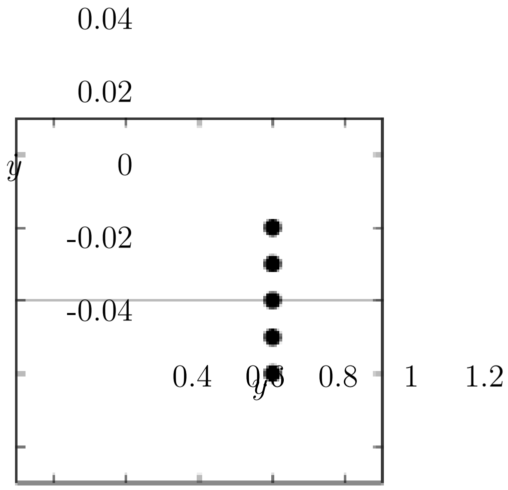
(c) n

(d) λ

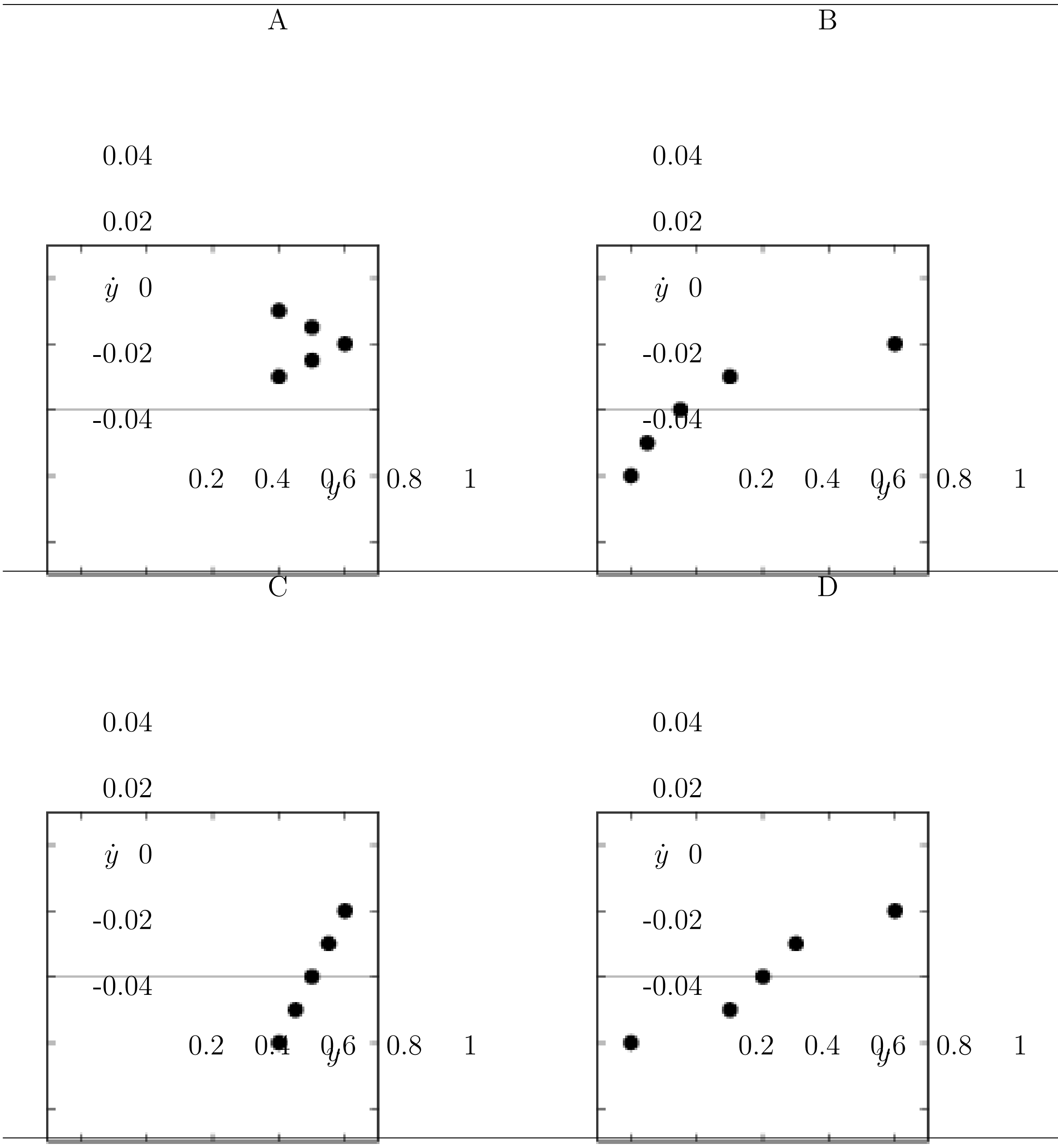
(e) What is the growth rate of per capita consumpion $c = \frac{C}{L}$ in the steady state in this model?

31. `[Problem ID #IDNO] convergence; definition, example`
Explain what is meant by *convergence* in growth theory. Compare Japan, South Korea, and North Korea as an example.

32. `[Problem ID #IDNO] convergence; graphical description`
Suppose that a group of countries recently showed the pattern in the graph below for *income growth rate vs. level of income.*



(a) Which of the following graphs represents the most likely state of convergence (or failure to converge) 30 years hence?



(b) Explain your answer.

33. `[Problem ID #IDNO] convergence; graphical description`
Each of the graphs in Fig. 1 shows a group of three countries, plotting their *per capita* income and their *per capita* income growth rate. One will display convergence of fast-growing low-income countries to the status of the high income country, the other will not.

   (a) Label each graph as "Converging" or "Not Converging".

(b) On each graph, plot the likely positions of the countries after ten years (remember, the highest income country always has the value 1 because income is measured as a fraction of that country's income).

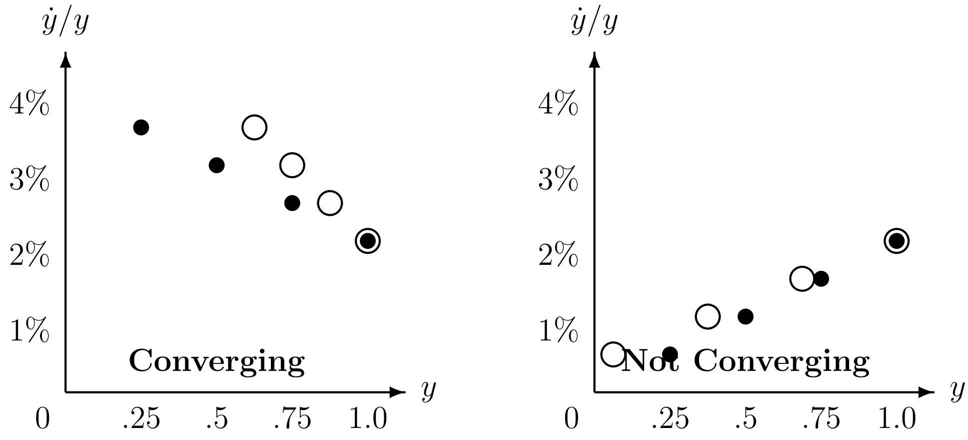


Figure 1: Convergence examples

Note: to get credit for the graph, you just need to put all the points to right of the original line of dots for the "Converging" case and to the left for the "Not Converging" case. For full credit, the highest income point should be the same before and after.

34. `[Problem ID #IDNO] steady state; vs equilibrium`
What is the difference between *steady state* and an *equilibrium*? (Keep your answer as simple as possible.)

In a steady state, only the state variable need be constant. Other variables can fluctuate. In equilibrium, none of the variables will have a tendency to change.

35. `[Problem ID #IDNO] steady state; vs equilibrium`
Why does dynamic analyis focus on *steady state* instead of *equilibrium*?

In the real world, with passage of time exogenous variables will change; it can't be helped. Steady state allows us to focus on economic relations that are in some sense "in balance" by "subtracting" (abstracting from) changes forced by exogenous variables.

36. `[Problem ID #IDNO] exponential growth; computation`
Assume constant savings rate of s in the Solow growth model, and a

TFP growth rate of 2%. After one generation (32 years) of balanced steady state growth, what is the per capita consumption?

37. `[Problem ID #IDNO] discounting; computation`
Suppose the interest rate is $\delta = \frac{1}{3}$ (33.3%). Compute the *present discounted value* of 100,000 yen received in two years' time.
The discount factor is $\frac{1}{1+\delta} = \frac{3}{4}$. *Thus the present discounted value is*
$$V = \left(\frac{1}{1+0.25}\right)^2 100000 = 56250\text{yen}.$$

38. `[Problem ID #IDNO] CRTS`
Show that the linear production function $Y = 3K + 4L$ is a *constant returns to scale* production function. (If you can't remember the mathematical proof, give three or four examples showing the property of constant returns to scale for partial credit.)

39. `[Problem ID #IDNO] CRTS`
Consider three different production functions F_a, F_b, and F_c. For each case i below, (1) answer whether the production function F_i appears to satisfy constant returns to scale (CRTS), and (2) justify your answer (*i.e.*, explain why or why not).
   (a) $F_a(12, 30) = 54$ and $F_a(4, 10) = 18$.
   This production function appears to be CRTS because $12 = 4\gamma$, $30 = 10\gamma$, *and* $54 = 18\gamma$, *where* $\gamma = 3$ *in each equation.*
   (b) $F_b(19, 6) = 32$ and $F_b(38, 12) = 48$.
   This production function cannot be CRTS because if $\gamma = 2$, *then* $38 = 19\gamma$ *and* $12 = 6\gamma$, *but* $48 \neq 64 = 32\gamma$.
   (c) $F_c(3, 4) = 5$ and $F_c(12, 8) = 15$.
   It is not possible to determine whether this function satisfies CRTS or not, because no two of the corresponding values are in the same ratio γ.

40. `[Problem ID #IDNO] history of thought`
Match each school of economics with the factor of production that it emphasizes as the source of growth or wealth, and the explanation of why the factor was considered most important by that school.
Please fill in your answers below, in lines 1–5. Please do not write out

the answers, but use the labels A–E.
Note that in the table of alternatives, each column is in alphabetical order, so there is no logical connection in the order here.

Label	**School**	**Factor**	**Explanation**
		Alternatives	
A	Ancient (Greeks, Romans)	Capital	History has many examples of countries getting rich by going to war and taking this factor from other countries.
B	Classical (Smith, Marx)	Labor	Not really a factor of production, the school that concentrated on this idea believed that wealth came from God and was concerned with “fairness.”
C	Neoclassical (Solow)	Land	This factor can be accumulated by allocating resources to it, and therefore is the natural focus of a dynamic theory of economic growth.
D	Physiocrat	Price	This factor is naturally productive alone, and there are limits to how much Man can improve its productivity.
E	Mediæval	Treasure	This human factor can be made more productive by specialization and trade, so its allocation is naturally the central issue of economics.
		Answers	
1	A	E	A
2	B	B	E
3	C	A	C
4	D	C	D
5	E	D	B